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Canonical Quantization and Chromodynamics in a Spherical Cavity

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The canonical quantization formalism is applied to the Lagrange density of chromodynamics, which includes gauge fixing and Faddeev–Popov ghost terms in a general covariant gauge. We develop the quantum theory of the interacting fields in the Dirac picture, based on the Gell-Mann and Low theorem and the Dyson expansion of the time evolution operator. The physical states are characterized by their invariance under Becchi–Rouet–Stora transformations. Subsequently, confinement is introduced phenomenologically by imposing, on the quark, gluon, and ghost field operators, the linear boundary conditions of the MIT bag model at the surface of a spherically symmetric and static cavity. Based on this formalism, we calculate, in the Feynman gauge, all nondivergent Feynman diagrams of second order in the strong coupling constant g. Explicit values of the matrix elements are given for low-lying quark and gluon cavity modes.

1. INTRODUCTION

During the last decade quantum chromodynamics (Yang and Mills, 1954; Fritzsch *et al.*, 1973; Gross and Wilczek, 1973a,b; Politzer, 1973) has emerged as the leading candidate for a theory of strong interactions of quarks and gluons. Due to asymptotic freedom (Gross and Wilczek, 1973a,b; Politzer, 1973), perturbative quantum chromodynamics has had considerable success in describing hadronic physics at high energies. In the low-energy region (below a few GeV), however, the running coupling constant

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$\alpha_s(Q^2)$ becomes large and nonperturbative effects (e.g., instantons, gluon condensation) cannot be neglected. The current belief is that this nonperturbative behavior, which is strongly enhanced by the non-Abelian character of the gauge group $SU(3)_{\text{color}}$, and still remains to be understood, leads eventually to the confinement of the color-carrying constituents in the hadron.

In the past, confinement has been incorporated into the theory using phenomenological models. The simplest of this kind is the MIT bag model (Chodos *et al.*, 1974a,b; DeGrand *et al.*, 1975; Johnson, 1975), in which boundary conditions are imposed on the quark and gluon fields at an arbitrary spacelike surface, preferably a static sphere. Of course, such a phenomenological approach to the theory of strong interaction is not very attractive, particularly because the confinement property is probably already contained in the theory as we perceive it today. Moreover, there are many other ways to achieve quark and gluon confinement (Bardeen *et al.*, 1975; Friedberg and Lee, 1977a,b, 1978; Vento *et al.*, 1980; Miller *et al.*, 1980; Théberge *et al.*, 1980; Thomas *et al.*, 1981). The advantage of imposing boundary conditions on the field operators, however, is that with a minimal number of modifications, we can incorporate confinement and at the same time retain most of the properties of the underlying gauge theory. Thus, the purpose of this paper is to show that it is indeed possible to formulate a consistent quantum field theory in such a cavity. The hope is, of course, that perturbative quantum chromodynamics in this cavity is a reasonable approximation to the real hadronic world.

In Section 2, we review the classical non-Abelian gauge theory (Yang and Mills, 1954; Fritzsch *et al.*, 1973). The local gauge invariance of chromodynamics is broken in a Lorentz-invariant way in order to obtain a local and well-defined Hamilton formulation of the theory that is suitable for quantization. Introducing the Faddeev–Popov (1967) ghost term into the Lagrange density, we enlarge the set of the elementary fields by two anticommuting real Grassmann fields that describe the unphysical ghosts. Subsequently, we discuss in some detail the so-called Becchi–Rouet–Stora (1974, 1976) (BRS) invariance of the modified theory. This global BRS invariance, which involves the ghost fields and replaces the broken local gauge symmetry, appears in a quite natural way in a modern geometrical formulation of the gauge theory. After a discussion of the classical field equations and the canonical conjugate momenta, we evaluate the Hamilton density and identify the various interaction terms. We conclude Section 2 with a review of the symmetries of chromodynamics. Here we do not attempt to describe all of the invariances of the theory, such as isospin symmetry for equal-mass quarks or chiral symmetry for massless quarks. We instead concentrate on the symmetries that are related to the gauge character of chromodynamics. Finally, the conserved current densities and the charges associated with these symmetries, in particular the Becchi-Rouet-Stora (BRS) and the ghost charge, are briefly discussed.

In Section 3, the theory is quantized using the canonical formalism, and the consistency of the quantization rules is checked by evaluating the commutation relations of the field operators with the Hamilton operator explicitly. As expected, these turn out to be consistent with the field equations and the definition of the canonical conjugate momenta. We also determine the commutators involving the Hamiltonian, the BRS charge, and the ghost number operator, and assure the nilpotency of the BRS charge. Subsequently, the field operators are transformed into the interaction picture in which the field operators satisfy the noninteracting field equations. Based on Dyson's expansion and the Gell-Mann and Low (1951) theorem, we derive the eigenvectors and eigenvalues of the full Hamilton operator as a perturbative expansion of the time evolution operator. We then turn to the definition of physical states that have nonnegative norm in order to guarantee a consistent probabilistic interpretation of the quantum field theory. Recognizing that the Gupta (1950)–Bleuler (1950) condition cannot be used in non-Abelian theories, we define the physical states as the ones that are BRS-invariant (Kugo and Ojima, 1979). In this physical subspace matrix elements of gluon field operators satisfy the generalized Maxwell equations. We also discuss briefly the physicality condition for the asymptotic states.

In Section 4, we impose on the field operators the linear boundary conditions of the MIT bag model (Chodos *et al.*, 1974a; Hansson and Jaffe, 1983; Goldhaber *et al.*, 1983, 1986), thus introducing confinement by hand. The boundary conditions turn out to be compatible with the field equations and the symmetries of quantum chromodynamics, including the BRS-invariance and the ghost charge conservation. Subsequently, the field operators are expanded in terms of the cavity modes of a spherically symmetric and static cavity, the expansion coefficients being the usual creation and annihilation operators for the various fields. Expressing the relevant part of the BRS charge in terms of these creation and annihilation operators, we find a close relationship between the Gupta–Bleuler condition and the physicality criterion for the asymptotic states we use here. Finally, we arrive at a representation of the noninteracting Hamilton operator that assures that only the physical degrees of freedom contribute to the energy of an asymptotic physical system.

In Section 5, we discuss cavity quantum chromodynamics up to second-order perturbation theory. On the example of the quark–quark interaction through one-gluon exchange, we illustrate the necessary steps in obtaining the energy shifts from the perturbative expansions given in Section 3. Subsequently, we discuss all nondivergent interactions for particles that occupy the lowest energy cavity modes: the quark–quark, antiquark–antiquark, and quark–antiquark interactions via one-gluon exchange and the quark–antiquark interaction through virtual annihilation into a gluon. We also evaluate the quark–gluon and antiquark–gluon interaction through the Compton diagram and one-gluon exchange and, finally, the gluon–gluon interaction mediated by the one-gluon exchange, the virtual annihilation into a gluon, and the elementary four-gluon vertex. The results are compared whenever possible with similar calculations of other groups. Finally, in Section 6, we draw some conclusions and discuss briefly the consistency of this quantum field theory.

In Appendix A, we determine the cavity modes that are used to expand the various field operators in Section 4. These cavity modes satisfy the classical, noninteracting ($g=0$) field equations of chromodynamics in the Feynman gauge together with the boundary conditions of Section 4. Appendix B contains a complete list of the Feynman propagators. In Appendix C, we evaluate the numerous integrals that arise at the various vertices and define the vertex functions. The bulk of the calculations of the two-particle energy shifts can be found in Appendix D. Here we discuss the two-body energy shift operators in first and second quantization for particles with arbitrary quantum numbers as well as the simplifications for low-energy cavity modes. Finally, Appendix E summarizes our conventions.

2. THE CLASSICAL THEORY

2.1. Introduction

In order to establish the notation, let us start with a brief introduction to chromodynamics, the classical field theory that is based on the non-Abelian gauge group $SU(3)_{\text{color}}$. This renormalizable field theory can be derived from the locally gauge invariant Lagrange density (Yang and Mills, 1954; Fritzsch *et al.*, 1973)

$$\mathscr{L}_{\text{gauge}} = \bar{\psi}(i\gamma_\mu D^\mu - M)\psi - \tfrac{1}{4}\mathbf{F}_{\mu\nu}\cdot\mathbf{F}^{\mu\nu} \tag{2.1}$$

The interactions between quarks and gluons are determined by the covariant derivative, which depends on the strong coupling constant g

$$D^\mu\psi = (\partial^\mu - ig\boldsymbol{\lambda}/2\cdot\mathbf{A}^\mu)\psi \tag{2.2}$$

and the gluon self-interaction originates from the term containing the chromoelectromagnetic field $\mathbf{F}^{\mu\nu}$, which can be expressed in terms of the real gluon fields $\mathbf{A}^\mu$ as

$$\mathbf{F}^{\mu\nu} = \partial^\mu\mathbf{A}^\nu - \partial^\nu\mathbf{A}^\mu + g\mathbf{A}^\mu\times\mathbf{A}^\nu \tag{2.3}$$

The antisymmetric field strength tensor $\mathbf{F}^{\mu\nu}$ and the covariant derivative D^{μ} are connected by the Bianchi identity

$$[D^{\mu}, D^{\nu}] = -\tfrac{1}{2} i g \boldsymbol{\lambda} \cdot \mathbf{F}^{\mu\nu} \tag{2.4}$$

In equations (2.1)-(2.4) we have made use of the eight-dimensional scalar and vector products operating in the color space of the gluons

$$\mathbf{A} \cdot \mathbf{B} = \sum_{a=1}^{8} A_a B_a \tag{2.5}$$

and

$$(\mathbf{A} \times \mathbf{B})_a = \sum_{b,b'=1}^{8} f_{abb'} A_b B_{b'} \tag{2.6}$$

respectively. The indices a, b, and b' describe the eight color degrees of freedom of the gluon, the $f_{abb'}$ being the structure constants of $SU(3)_{\text{color}}$, and the λ_a denote the eight Gell-Mann matrices.

The symbol ψ denotes a large column consisting of the complex quark fields $\psi_{cf\alpha}$, where the labels c, f, and α stand for the various color ($c = 1, 2, 3$), flavor ($f = 1, \ldots, 6$), and Dirac ($\alpha = 1, 2, 3, 4$) indices. The mass matrix M is diagonal in these labels and depends only on the flavor label of the quarks, i.e.,

$$M_{c,f,\alpha;c',f',\alpha'} = \delta_{cc'} \delta_{ff'} \delta_{\alpha\alpha'} m_f \tag{2.7}$$

where m_f denotes the mass of the quark with flavor f.

The Lagrange density (2.1) is, by construction, invariant under infinitesimal local gauge transformation of the fields ψ and $\mathbf{A}^{\mu}$,

$$\begin{aligned} \psi &\to \psi' = (1 - \tfrac{1}{2} i \varepsilon \boldsymbol{\lambda} \cdot \boldsymbol{\omega}) \psi \\ \mathbf{A}_{\mu} &\to \mathbf{A}'_{\mu} = \mathbf{A}_{\mu} - \varepsilon \mathbf{A}_{\mu} \times \boldsymbol{\omega} - (\varepsilon / g)\, \partial_{\mu} \boldsymbol{\omega} \end{aligned} \tag{2.8}$$

where ε denotes an infinitesimal, real, constant parameter and $\boldsymbol{\omega}$ is a real, space-time-dependent function.

2.2. Gauge Fixing and BRS Invariance

It is well known that the Lagrange density (2.1) is not suitable for the quantization of the classical gauge theory. This is intimately related to the gauge freedom (2.8), as can be seen either in the path integral formalism or, in the case of canonical quantization, by the vanishing canonical conjugate momentum $\mathbf{\Pi}^0$ of $\mathbf{A}^0$. In order to obtain a consistent quantum theory, we must choose a gauge and therefore break the local gauge invariance (2.8) of the Lagrange density (2.1). The standard approach is to add to (2.1) a covariant gauge-fixing term which is globally gauge invariant,

$$\mathscr{L}_{\text{fix}} = -\tfrac{1}{2} \lambda\, \partial_{\mu} \mathbf{A}^{\mu} \cdot \partial_{\nu} \mathbf{A}^{\nu} \tag{2.9}$$

Here λ is a real parameter characterizing the gauge. The broken local gauge invariance can be substituted by the so-called Becchi-Rouet-Stora (1974, 1976) (BRS) invariance, which is usually introduced with the help of path integrals. The following derivation may seem more elementary.

Under a local gauge transformation (2.8) the variations of $\mathbf{A}_\mu$ and $\mathscr{L}_{\text{fix}}$ are

$$\begin{aligned}\delta_\varepsilon \mathbf{A}_\mu &= -(\varepsilon/g)\mathscr{D}_\mu \boldsymbol{\omega} \\ \delta_\varepsilon \mathscr{L}_{\text{fix}} &= (\lambda\varepsilon/g)\,\partial_\mu \mathbf{A}^\mu \cdot \partial_\nu \mathscr{D}^\nu \boldsymbol{\omega}\end{aligned} \tag{2.10}$$

where we have introduced the covariant derivative in the adjoint representation of the gauge group

$$\mathscr{D}_\mu \boldsymbol{\omega} = \partial_\mu \boldsymbol{\omega} + g\mathbf{A}_\mu \times \boldsymbol{\omega} \tag{2.11}$$

Thus, if we constrain the gauge phases $\boldsymbol{\omega}$ to satisfy

$$\partial_\mu \mathscr{D}^\mu \boldsymbol{\omega} = 0 \tag{2.12}$$

$\mathscr{L}_{\text{fix}}$ is indeed invariant. This constraint can easily be included in the Lagrange density using the well-known method of Lagrange multipliers. Thus, by adding the so-called Faddeev-Popov ghost term

$$\mathscr{L}_{\text{ghost}} = i\boldsymbol{\chi} \cdot \partial_\mu \mathscr{D}^\mu \boldsymbol{\omega} \tag{2.13}$$

we arrive at the total Lagrange density

$$\mathscr{L}' = \mathscr{L}_{\text{gauge}} + \mathscr{L}_{\text{fix}} + \mathscr{L}_{\text{ghost}} \tag{2.14}$$

In equation (2.13) the Lagrange multipliers for the subsidiary condition (2.12) have been denoted by $i\boldsymbol{\chi}$.

Of course, the additional terms $\mathscr{L}_{\text{fix}}$ and $\mathscr{L}_{\text{ghost}}$ change the physical properties of the system we want to describe. In a consistent theory, every field that appears in the Lagrange density must be a dynamical field, giving rise to the corresponding particles in the quantum version of the theory. Hence, the Lagrange density (2.14) describes a theory of quarks, gluons, and the new $\boldsymbol{\omega}$ and $\boldsymbol{\chi}$ ghosts. A similar situation occurred in the transition from global to local gauge invariance, where the gauge bosons appeared as new particles.

We can now design a transformation of the fields ψ, $\mathbf{A}_\mu$, $\boldsymbol{\omega}$, and $\boldsymbol{\chi}$ that leaves the Lagrange density (2.14) invariant. Demanding that it resembles an infinitesimal local gauge transformation for the quark and gluon fields, we are led to

$$\begin{aligned}\delta_\varepsilon \psi &= -\tfrac{1}{2} i\varepsilon \boldsymbol{\lambda} \cdot \boldsymbol{\omega}\psi \\ \delta_\varepsilon \mathbf{A}_\mu &= -(\varepsilon/g)\mathscr{D}_\mu \boldsymbol{\omega}\end{aligned} \tag{2.15}$$

This transformation leaves $\mathcal{L}_{\text{gauge}}$ of equation (2.1) invariant. Moreover, since equation (2.12) is one of the field equations that can be derived from the Lagrange density (2.14), it must be invariant as well,

$$\delta_\varepsilon(\partial_\mu \mathscr{D}^\mu \boldsymbol{\omega}) = \partial_\mu(\delta_\varepsilon \mathscr{D}^\mu \boldsymbol{\omega}) = 0 \tag{2.16}$$

We thus conclude that $\delta_\varepsilon \mathcal{L}' = 0$ implies

$$\delta_\varepsilon \boldsymbol{\chi} = i\varepsilon(\lambda/g)\, \partial_\mu \mathbf{A}^\mu \tag{2.17}$$

The crucial step is now to define the variation of the field $\boldsymbol{\omega}$ such as to satisfy equation (2.16). We will in fact demand the even stronger condition

$$0 = \delta_\varepsilon \mathscr{D}_\mu \boldsymbol{\omega} = \mathscr{D}_\mu(\delta_\varepsilon \boldsymbol{\omega}) - (\varepsilon \mathscr{D}_\mu \boldsymbol{\omega}) \times \boldsymbol{\omega} \tag{2.18}$$

The second part of equation (2.18) is a consequence of the gluon field variation (2.15) and the definition of the covariant derivative (2.11). Interpreting equation (2.18) as a differential equation for $\delta_\varepsilon \boldsymbol{\omega}$ and solving it with standard methods yields a nonlocal result for $\delta_\varepsilon \boldsymbol{\omega}$. This unpleasant feature can be avoided if we take the parameter ε and the components of $\boldsymbol{\omega}$ in the classical theory to be anticommuting numbers.[4] In this case, equation (2.18) can be rewritten as

$$\mathscr{D}_\mu \delta_\varepsilon \boldsymbol{\omega} = \varepsilon(\mathscr{D}_\mu \boldsymbol{\omega}) \times \boldsymbol{\omega} = \tfrac{1}{2}\varepsilon \mathscr{D}_\mu(\boldsymbol{\omega} \times \boldsymbol{\omega}) \tag{2.19}$$

A simple and local solution of this equation takes the form

$$\delta_\varepsilon \boldsymbol{\omega} = \tfrac{1}{2}\varepsilon\, \boldsymbol{\omega} \times \boldsymbol{\omega} \tag{2.20}$$

where, due to the anticommutativity, the cross product $\boldsymbol{\omega} \times \boldsymbol{\omega}$ does not vanish.

2.3. Lagrange and Hamilton Densities

Let us now introduce a new Lagrange density $\mathcal{L}$ that differs from equation (2.14) by a four-divergence that does not contribute to the field equations,

$$\begin{aligned} \mathcal{L} = {} & \bar{\psi}(i\gamma_\mu D^\mu - M)\psi - \tfrac{1}{2} i\, \partial_\mu(\bar{\psi}\gamma^\mu \psi) - 1/4 \mathbf{F}_{\mu\nu} \cdot \mathbf{F}^{\mu\nu} \\ & - \tfrac{1}{2}\lambda\, \partial_\mu \mathbf{A}^\mu \cdot \partial_\nu \mathbf{A}^\nu - i\partial_\mu \boldsymbol{\chi} \cdot \mathscr{D}^\mu \boldsymbol{\omega} \end{aligned} \tag{2.21}$$

[4]One is reminded of a similar situation in the "derivation of the Dirac equation." There, it was possible to replace the nonlocal square root of the Klein–Gordon operator "$(\Box + m^2)^{1/2}$" by the local Dirac operator $i\gamma_\mu \partial^\mu - m$ with the help of the γ matrices, which satisfy anticommutation relations.

The BRS transformation of the fields, which we have found to be

$$\begin{aligned} \delta_\varepsilon\psi &= -\tfrac{1}{2}i\varepsilon\boldsymbol{\lambda}\cdot\boldsymbol{\omega}\psi \\ \delta_\varepsilon\mathbf{A}_\mu &= -(\varepsilon/g)\mathscr{D}_\mu\boldsymbol{\omega} \\ \delta_\varepsilon\boldsymbol{\omega} &= \tfrac{1}{2}\varepsilon\boldsymbol{\omega}\times\boldsymbol{\omega} \\ \delta_\varepsilon\boldsymbol{\chi} &= i\varepsilon(\lambda/g)\,\partial_\mu\mathbf{A}^\mu \end{aligned} \tag{2.22}$$

changes $\mathscr{L}$ by a four-divergence

$$\delta_\varepsilon\mathscr{L} = \varepsilon(\lambda/g)\,\partial_\mu[\partial_\nu\mathbf{A}^\mu\cdot\mathscr{D}^\mu\boldsymbol{\omega}] \tag{2.23}$$

that leaves the field equations invariant. Thus, the BRS invariance of the action can be used to replace the broken local gauge invariance. Even though it is a global symmetry (ε is space–time independent), it is sufficient to guarantee the validity of the Ward–Slavnov–Taylor identities (Slavnov, 1972; Taylor, 1971) that are necessary for the renormalizability ('tHooft, 1971a,b; Becchi *et al.*, 1974, 1976) of the corresponding quantum theory. The BRS parameter ε and the Faddeev–Popov ghost fields $\boldsymbol{\omega}$ and $\boldsymbol{\chi}$ are Grassmann numbers, satisfying the anticommutation relations

$$\begin{aligned} \{\varepsilon, \varepsilon\} = \{\varepsilon, \omega_a\} = \{\varepsilon, \chi_a\} = 0 \\ \{\omega_a, \omega_b\} = \{\omega_a, \chi_b\} = \{\chi_a, \chi_b\} = 0 \end{aligned} \tag{2.24}$$

Moreover, these fields commute with the real gluon fields A_μ^a and anticommute with the quark fields $\psi_{cf\alpha}$, which, for consistency, must be assumed to be complex Grassmann fields as well. The factor i in front of the ghost term in (2.21) ensures that the Lagrange density $\mathscr{L}$ is real in the classical theory (corresponding to a Hermitian Lagrange density in the quantum theory), if we use the convention (Kugo and Ojima, 1979)

$$\omega_a^* = \omega_a, \qquad \chi_a^* = \chi_a \qquad (\omega_a\chi_b)^* = \chi_b^*\omega_a^* = -\omega_a\chi_b \tag{2.25}$$

i.e., ω_a and χ_a are real Grassmann numbers. The consistency of the BRS transformation with the rules for complex conjugation of Grassmann variables requires the parameter ε in (2.22) to be an imaginary Grassmann number

$$\varepsilon^* = -\varepsilon, \qquad (\varepsilon\omega_a)^* = \omega_a^*\varepsilon^* = \varepsilon\omega_a \tag{2.26}$$

An important property of the BRS transformation, worth noticing already at this stage, is its nilpotency. Using the relations (2.22), it is easily verified that for any field ϕ

$$\delta_{\varepsilon_1}(\delta_{\varepsilon_2}\phi) = 0 \tag{2.27}$$

even though the product $\varepsilon_1\varepsilon_2$ does not vanish in general ($\varepsilon_1 \neq \varepsilon_2$).

The Lagrange density (2.21) can be separated into a g-independent or "free" part, which describes the "free" quark, gluon, and ghost fields

$$\mathscr{L}_0(\phi_i, \partial_\mu \phi_i) = \bar{\psi}(\tfrac{1}{2} i\gamma_\mu \overleftrightarrow{\partial}^\mu - M)\psi - \tfrac{1}{4}(\partial_\mu \mathbf{A}_\nu - \partial_\nu \mathbf{A}_\mu)\cdot(\partial^\mu \mathbf{A}^\nu - \partial^\nu \mathbf{A}^\mu)$$
$$-\tfrac{1}{2}\lambda\, \partial_\mu \mathbf{A}^\mu \cdot \partial_\nu \mathbf{A}^\nu - i\partial_\mu \boldsymbol{\chi}\cdot\partial^\mu \boldsymbol{\omega} \tag{2.28}$$

with $\overleftrightarrow{\partial}^\mu = \overrightarrow{\partial}^\mu - \overleftarrow{\partial}^\mu$, and a g-dependent or "interaction" part

$$\mathscr{L}_{\text{int}}(\phi_i, \partial_\mu \phi_i) = \tfrac{1}{2} g\bar{\psi}\gamma_\mu \boldsymbol{\lambda}\psi \cdot \mathbf{A}^\mu - \tfrac{1}{2} g(\partial^\mu \mathbf{A}^\nu - \partial^\nu \mathbf{A}^\mu)\cdot(\mathbf{A}_\mu \times \mathbf{A}_\nu)$$
$$-\tfrac{1}{4} g^2(\mathbf{A}^\mu \times \mathbf{A}^\nu)\cdot(\mathbf{A}_\mu \times \mathbf{A}_\nu) - ig\,\partial_\mu \boldsymbol{\chi}\cdot(\mathbf{A}^\mu \times \boldsymbol{\omega}) \tag{2.29}$$

which describes respectively the two-quark-one-gluon, three-gluon, four-gluon, and two-ghost-one-gluon couplings.

By applying the Euler–Lagrange equations to the Lagrange density (2.21), we readily arrive at the Dirac equations for the quark fields

$$\begin{aligned}(i\gamma_\mu D^\mu - M)\psi &= 0\\ \bar{\psi}(i\gamma_\mu \overleftarrow{D}^{\mu *} + M) &= 0\end{aligned} \tag{2.30}$$

and for the corresponding field equations for the gluon field we obtain

$$\mathscr{D}_\nu \mathbf{F}^{\nu\mu} + \tfrac{1}{2} g\bar{\psi}\gamma^\mu \boldsymbol{\lambda}\psi + \lambda\partial^\mu\, \partial_\nu \mathbf{A}^\nu + ig\,\partial^\mu \boldsymbol{\chi}\times\boldsymbol{\omega} = 0 \tag{2.31}$$

where the last two terms originate from the gauge fixing and ghost terms, respectively. By varying the Lagrange density (2.21) with respect to the ghost fields, we arrive at the field equations for the ghost fields

$$\partial_\mu \mathscr{D}^\mu \boldsymbol{\omega} = 0 \tag{2.32}$$

$$\mathscr{D}_\mu \partial^\mu \boldsymbol{\chi} = 0 \tag{2.33}$$

In order to write down the Hamilton density, we need to evaluate the canonical conjugate momenta of the interacting quark, gluon, and ghost fields. For the quark fields we readily obtain

$$\begin{aligned}\pi &= \partial\mathscr{L}/\partial\dot{\psi} = -\tfrac{1}{2} i\psi^+\\ \bar{\pi} &= \partial\mathscr{L}/\partial\dot{\bar{\psi}} = -\tfrac{1}{2} i\bar{\psi}^+\end{aligned} \tag{2.34}$$

and for the gluon fields we arrive at

$$\begin{aligned}\boldsymbol{\Pi}^k &= \partial\mathscr{L}/\partial\dot{\mathbf{A}}_k = \mathbf{F}^{k0}, \qquad k = 1, 2, 3\\ \boldsymbol{\Pi}^0 &= \partial\mathscr{L}/\partial\dot{\mathbf{A}}_0 = -\lambda\partial_\nu \mathbf{A}^\nu\end{aligned} \tag{2.35}$$

Thus, as we mentioned earlier, without the gauge-fixing term present in the Lagrange density (2.21), the zeroth component of the canonical conjugate momentum would vanish and eventually lead to an ill-defined Hamilton density. Moreover, the addition of the four-divergence involving the quark fields in equation (2.21) was necessary to obtain a nonvanishing canonical conjugate momentum $\bar{\pi}$. The canonical conjugate momenta of the ghost fields are given by

$$\begin{aligned} \mathbf{X} &= \partial \mathscr{L}/\partial \dot{\boldsymbol{\chi}} = -i\mathscr{D}_0 \boldsymbol{\omega} \\ \boldsymbol{\Omega} &= \partial \mathscr{L}/\partial \dot{\boldsymbol{\omega}} = i\dot{\boldsymbol{\chi}} \end{aligned} \tag{2.36}$$

The positive sign in the last equation results from the anticommuting character of the Grassmann fields ω_a and χ_a and the definition of the derivative with respect to a Grassmann field. The dependence of the conjugate momenta on the coupling constant g in equations (2.35) and (2.36) arises from the derivative couplings in the original Lagrange density and is often a major source of confusion.

We now turn to the evaluation of the Hamilton density, which is defined as

$$\mathscr{H} = -\frac{\partial \mathscr{L}}{\partial \dot{\psi}} \dot{\psi} + \dot{\bar{\psi}} \frac{\partial \mathscr{L}}{\partial \dot{\bar{\psi}}} + \dot{\mathbf{A}}_\mu \cdot \frac{\partial \mathscr{L}}{\partial \dot{\mathbf{A}}} + \dot{\boldsymbol{\chi}} \cdot \frac{\partial \mathscr{L}}{\partial \dot{\boldsymbol{\chi}}} + \dot{\boldsymbol{\omega}} \cdot \frac{\partial \mathscr{L}}{\partial \dot{\boldsymbol{\omega}}} - \mathscr{L} \tag{2.37}$$

where the negative sign in the first term is due to the Grassmann character of the quark fields. $\mathscr{H}$ is a function of the fields, the spatial derivatives, and the corresponding canonical momenta; it no longer depends on the time derivatives of the fields. We must thus replace all time derivatives of the fields by the corresponding canonical momenta, some of which will depend on the strong coupling constant g. The Hamilton density can now be split into a part that does not depend on the coupling constant g explicitly

$$\begin{aligned} \mathscr{H}_0 = {} & \bar{\psi}(-\tfrac{1}{2} i\gamma_k \overleftrightarrow{\partial}^k + M)\psi + \tfrac{1}{4}(\partial_k \mathbf{A}^l - \partial_l \mathbf{A}^k) \cdot (\partial_k \mathbf{A}^l - \partial_l \mathbf{A}^k) + \tfrac{1}{2}\boldsymbol{\Pi}^k \cdot \boldsymbol{\Pi}^k \\ & - (1/2\lambda)\boldsymbol{\Pi}^0 \cdot \boldsymbol{\Pi}^0 + \boldsymbol{\Pi}^k \cdot \partial_k \mathbf{A}^0 - \boldsymbol{\Pi}^0 \cdot \partial_k \mathbf{A}^k - i\boldsymbol{\Omega} \cdot \mathbf{X} - i\partial_k \boldsymbol{\chi} \cdot \partial_k \boldsymbol{\omega} \end{aligned} \tag{2.38}$$

and an interaction term that depends linearly and quadratically on the coupling constant g

$$\begin{aligned} \mathscr{H}_{\text{int}} = {} & -\tfrac{1}{2} g\bar{\psi}\gamma_\mu \boldsymbol{\lambda}\psi \cdot \mathbf{A}^\mu - \tfrac{1}{2} g(\partial_k \mathbf{A}^l - \partial_l \mathbf{A}^k) \cdot (\mathbf{A}^k \times \mathbf{A}^l) \\ & - g\boldsymbol{\Pi}^k \cdot (\mathbf{A}^k \times \mathbf{A}^0) + \tfrac{1}{4} g^2 (\mathbf{A}^k \times \mathbf{A}^l) \cdot (\mathbf{A}^k \times \mathbf{A}^l) \\ & + g\boldsymbol{\Omega} \cdot (\mathbf{A}^0 \times \boldsymbol{\omega}) + ig\partial_k \boldsymbol{\chi} \cdot (\mathbf{A}^k \times \boldsymbol{\omega}) \end{aligned} \tag{2.39}$$

The various terms in equation (2.39) describe respectively the two-quark-one-gluon, three-gluon, four-gluon, and two-ghost-one-gluon interactions (Figure 1). Note that, due to the derivative couplings in the Lagrange density

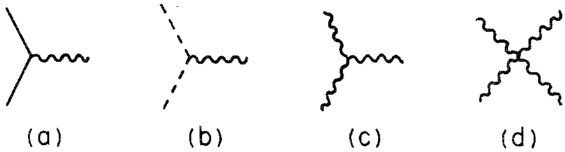


Fig. 1. The vertices describing the emission (or absorption) of a gluon by (a) a quark, (b) a ghost, and (c) a gluon and (d) the elementary four-gluon vertex.

(2.29), $\mathscr{H}_{\text{int}}$ differs from $-\mathscr{L}_{\text{int}}$, and we have

$$\mathscr{H}_{\text{int}} = -\mathscr{L}_{\text{int}}(\phi_i, \partial_\mu \phi_i) - \tfrac{1}{2}g^2(\mathbf{A}^0 \times \mathbf{A}^k)\cdot(\mathbf{A}^0 \times \mathbf{A}^k) \tag{2.40}$$

2.4. Conserved Currents

Let us now briefly discuss some of the invariances and conservation laws of the theory given by the Lagrange density (2.21). The global Abelian phase transformation of the quark fields alone leads to the conserved Noether current

$$J^\mu_Q = \bar\psi\gamma^\mu\psi; \qquad \partial_\mu J^\mu_Q = 0 \tag{2.41}$$

thus ensuring the conservation of the quark number. The color-carrying quark current, however, is not conserved; instead, it is conserved covariantly, in the sense of the gauge group

$$\mathbf{J}^\mu_Q = \tfrac{1}{2}\bar\psi\gamma^\mu\boldsymbol{\lambda}\psi; \qquad \mathscr{D}_\mu \mathbf{J}^\mu_Q = 0 \tag{2.42}$$

The nonconservation of $\mathbf{J}^\mu_Q$ reflects the fact that quarks are not the only color-carrying particles of the theory.

Even though the Lagrange density (2.21) is not invariant under a local gauge transformation, a global (space–time-independent) gauge transformation is still a good symmetry of the theory. The corresponding Noether current reads

$$\begin{aligned}\mathbf{J}^\mu_C &= \tfrac{1}{2}\bar\psi\gamma^\mu\boldsymbol{\lambda}\psi + \mathbf{F}^{\mu\nu}\times\mathbf{A}_\nu + \lambda\partial_\nu\mathbf{A}^\nu\times\mathbf{A}^\mu + i\partial^\mu\boldsymbol{\chi}\times\boldsymbol{\omega} - i\boldsymbol{\chi}\times\mathscr{D}^\mu\boldsymbol{\omega}\\ \partial_\mu\mathbf{J}^\mu_C &= 0\end{aligned} \tag{2.43}$$

and leads to color conservation. As we have demonstrated above, the BRS transformation leaves the action of chromodynamics invariant. Taking due account of equation (2.23), the associated current is easily found to be

$$\begin{aligned}J^\mu_B &= \boldsymbol{\omega}\cdot(\mathscr{D}_\nu\mathbf{F}^{\nu\mu} + \tfrac{1}{2}g\bar\psi\gamma^\mu\boldsymbol{\lambda}\psi + \tfrac{1}{2}ig\partial^\mu\boldsymbol{\chi}\times\boldsymbol{\omega}) + \lambda\mathscr{D}^\mu\boldsymbol{\omega}\cdot\partial_\nu\mathbf{A}^\nu\\ \partial_\mu J^\mu_B &= 0\end{aligned} \tag{2.44}$$

Here, we have omitted a term of the form $\partial_\nu(\boldsymbol{\omega}\cdot\mathbf{F}^{\mu\nu})$, which is conserved by itself and does not contribute to the charge upon integration.[5] The resulting conserved charge

$$Q_B=\int d^3x\,J_B^0=\int d^3x\,[\boldsymbol{\omega}\cdot(\mathscr{D}_k\boldsymbol{\Pi}^k+\tfrac{1}{2}g\psi^+\boldsymbol{\lambda}\psi+\tfrac{1}{2}g\boldsymbol{\Omega}\times\boldsymbol{\omega})-i\mathbf{X}\cdot\boldsymbol{\Pi}^0] \quad (2.45)$$

is a real Grassmann number, called the BRS charge; it is of great importance in the quantized theory.

Finally, the Lagrange density (2.21) is invariant under a symmetry transformation that involves the ghost fields $\boldsymbol{\omega}$ and $\boldsymbol{\chi}$ only, similar to the Abelian phase transformation of the quark fields. Of course, since the ghost fields are real Grassmann numbers, we are not allowed to perform a complex phase transformation upon them. However, the following scale transformation is admissible (Kugo and Ojima, 1979):

$$\boldsymbol{\omega}\to e^{\Theta}\boldsymbol{\omega},\qquad \boldsymbol{\chi}\to e^{-\Theta}\boldsymbol{\chi} \quad (2.46)$$

where Θ is a real, space-time-independent number. As a consequence, we find

$$J_G^\mu=i\partial^\mu\boldsymbol{\chi}\cdot\boldsymbol{\omega}-i\boldsymbol{\chi}\cdot\mathscr{D}^\mu\boldsymbol{\omega};\qquad \partial_\mu J_G^\mu=0 \quad (2.47)$$

giving rise to the conservation of the ghost charge

$$Q_G=\int d^3x\,J_G^0=\int d^3x\,(\boldsymbol{\Omega}\cdot\boldsymbol{\omega}+\boldsymbol{\chi}\cdot\mathbf{X}) \quad (2.48)$$

3. THE QUANTIZED THEORY

3.1. Canonical Quantization

In the preceding section, we developed a Hamiltonian formulation of chromodynamics that is suitable for the quantization of the theory. Rather than relying on the usual path integral methods, we want to apply the canonical quantization formalism to the Hamilton densities $\mathscr{H}_0$ and $\mathscr{H}_{\text{int}}$ as given in equations (2.38) and (2.39). We thus impose the equal-time anticommutation and commutation relations for the quark and the Hermitian gluon

[5]Even though $\partial_\nu(\boldsymbol{\omega}\cdot\mathbf{F}^{\mu\nu})$ is conserved due to the antisymmetry of $\mathbf{F}^{\mu\nu}$, it does not necessarily lead to a well-defined charge in the quantized theory. In the general case, it should therefore be kept in the BRS current in order to obtain a well-defined BRS charge. The existence of this charge is assured by the assumption that the BRS symmetry remains unbroken in the quantized theory. However, in view of the boundary conditions (4.8), the charge corresponding to $\partial_\nu(\boldsymbol{\omega}\cdot\mathbf{F}^{\mu\nu})$ is well defined and integrates to zero in the finite-volume theory.

field operators, respectively:

$$\{\psi_{cf\alpha}(\mathbf{x}, t), \psi^+_{c'f'\alpha'}(\mathbf{y}, t)\} = \delta_{cc'}\delta_{ff'}\delta_{\alpha\alpha'}\delta^{(3)}(\mathbf{x}-\mathbf{y}) \tag{3.1}$$

$$[A^\mu_a(\mathbf{x}, t), \Pi^\nu_b(\mathbf{y}, t)] = ig^{\mu\nu}\delta_{ab}\delta^{(3)}(\mathbf{x}-\mathbf{y}) \tag{3.2}$$

The Hermitian ghost field operators must satisfy anticommutation relations, since they are described in the classical theory by real, anticommuting color-octet and spin-zero Grassmann fields

$$\{\omega_a(\mathbf{x}, t), \Omega_b(\mathbf{y}, t)\} = -i\delta_{ab}\delta^{(3)}(\mathbf{x}-\mathbf{y}) \tag{3.3}$$

$$\{\chi_a(\mathbf{x}, t), X_b(\mathbf{y}, t)\} = -i\delta_{ab}\delta^{(3)}(\mathbf{x}-\mathbf{y}) \tag{3.4}$$

Here it is understood that all commutators of the gluon field operators, all anticommutators of the quark and ghost field operators, and all commutators involving gluon and quark or ghost field operators that have not been written down explicitly vanish.

As a consistency check, we can evaluate the commutation relations of the field operators and the corresponding canonical momenta with the Hamilton operator defined as

$$H = H_0(t) + H_{\text{int}}(t) = \int d^3x\, \mathcal{H}_0(\mathbf{x}, t) + \int d^3x\, \mathcal{H}_{\text{int}}(\mathbf{x}, t) \tag{3.5}$$

Here $\mathcal{H}_0(\mathbf{x}, t)$ and $\mathcal{H}_{\text{int}}(\mathbf{x}, t)$ are formally given by (2.38) and (2.39), but now the fields must be interpreted as field operators. If the quantization rules are correct, these commutators and anticommutators must yield Heisenberg equations of motion that are equivalent to the Euler-Lagrange equations for the field operators. Indeed, using the anticommutation relations (3.1), the commutators for the quark field operators turn out to be

$$[\psi, H] = \gamma^0(-i\gamma^k\partial_k\psi + M\psi - \tfrac{1}{2}g\gamma^\mu\mathbf{A}_\mu\cdot\boldsymbol{\lambda}\psi) = i\dot{\psi} \tag{3.6}$$

$$[\psi^+, H] = -i\partial_k\bar{\psi}\gamma^k - \bar{\psi}M + \tfrac{1}{2}g\bar{\psi}\gamma^\mu\mathbf{A}_\mu\cdot\boldsymbol{\lambda} = i\dot{\psi}^+ \tag{3.7}$$

which are equivalent to the Dirac equations (2.30). Based on the commutation relations for the gluon field operators (3.2), we arrive at

$$[\mathbf{A}^0, H] = i[-\partial_k\mathbf{A}^k - (1/\lambda)\mathbf{\Pi}^0] = i\dot{\mathbf{A}}^0 \tag{3.8}$$

$$[\mathbf{A}^k, H] = i(-\mathbf{\Pi}^k - \partial_k\mathbf{A}^0 + g\mathbf{A}^k\times\mathbf{A}^0) = i\dot{\mathbf{A}}^k \tag{3.9}$$

consistent with the definition of the canonical conjugate momenta (2.35). Moreover, the commutators of the canonical momenta with H yield

$$[\mathbf{\Pi}^0, H] = i(-\mathcal{D}^k\mathbf{\Pi}^k + \tfrac{1}{2}g\bar{\psi}\gamma^0\boldsymbol{\lambda}\psi + g\mathbf{\Omega}\times\boldsymbol{\omega}) = i\dot{\mathbf{\Pi}}^0 \tag{3.10}$$

$$[\mathbf{\Pi}^k, H] = i(\mathcal{D}^l\mathbf{F}^{kl} + \partial_k\mathbf{\Pi}^0 - g\mathbf{A}^0\times\mathbf{\Pi}^k + \tfrac{1}{2}g\bar{\psi}\gamma^k\boldsymbol{\lambda}\psi - ig\partial_k\boldsymbol{\chi}\times\boldsymbol{\omega}) = i\dot{\mathbf{\Pi}}^k \tag{3.11}$$

which are equivalent to the field equations for the gluons as given in equations (2.31). Finally, using the anticommutators (3.3) and (3.4), the ghost field operators are easily shown to satisfy

$$[\boldsymbol{\omega}, H] = -\mathbf{X} - ig\mathbf{A}^0 \times \boldsymbol{\omega} = i\dot{\boldsymbol{\omega}} \tag{3.12}$$

$$[\boldsymbol{\chi}, H] = \boldsymbol{\Omega} = i\dot{\boldsymbol{\chi}} \tag{3.13}$$

consistent with the definition of the canonical conjugate momenta (2.36). Similarly, the commutators of the canonical momenta with the Hamilton operator turn out to be

$$[\boldsymbol{\Omega}, H] = \mathscr{D}_k \partial^k \boldsymbol{\chi} + ig\boldsymbol{\Omega} \times \mathbf{A}^0 = i\dot{\boldsymbol{\Omega}} \tag{3.14}$$

$$[\mathbf{X}, H] = -\partial_k \mathscr{D}^k \boldsymbol{\omega} = i\dot{\mathbf{X}} \tag{3.15}$$

which are equivalent to the field equations for the ghosts as given in equations (2.32) and (2.33).

The BRS transformation (2.22) can be described in the quantized theory with the help of the corresponding generator, the BRS charge, (2.45). Using the canonical commutation and anticommutation relations (3.1)-(3.4), we can show that

$$[i\varepsilon Q_B, F] = g\delta_\varepsilon F \tag{3.16}$$

for any field operator F, where $\delta_\varepsilon F$ is given by equations (2.22). Note that ε anticommutes with the operators ψ, $\boldsymbol{\omega}$, and $\boldsymbol{\chi}$.

In analogy with the decomposition of the Hamiltonian (3.35), we can separate Q_B into

$$Q_B = Q_0(t) + Q_{\text{int}}(t) \tag{3.17}$$

Here, $Q_0(t)$ is independent of the strong coupling constant g,

$$Q_0(t) = \int d^3x\, (\boldsymbol{\omega} \cdot \partial_k \boldsymbol{\Pi}^k - i\mathbf{X} \cdot \boldsymbol{\Pi}^0) \tag{3.18}$$

and $Q_{\text{int}}(t)$ is proportional to g,

$$Q_{\text{int}}(t) = g \int d^3x\, \boldsymbol{\omega} \cdot (\mathbf{A}_k \times \boldsymbol{\Pi}^k + \tfrac{1}{2}\psi^+ \boldsymbol{\lambda} \psi + \tfrac{1}{2}\boldsymbol{\Omega} \times \boldsymbol{\omega}) \tag{3.19}$$

The nilpotency (2.27) of the BRS transformation is now reflected in the anticommutators

$$\begin{aligned} \{Q_0(t), Q_0(t)\} &= 0 \\ \{Q_0(t), Q_{\text{int}}(t)\} &= 0 \\ \{Q_{\text{int}}(t), Q_{\text{int}}(t)\} &= 0 \\ \{Q_B, Q_B\} &= 0 \end{aligned} \tag{3.20}$$

whereas the commutators with the Hamiltonians

$$\begin{aligned} [Q_0(t), H_0(t)] &= 0 \\ [Q_0(t), H_{\text{int}}(t)] &= -[Q_{\text{int}}(t), H_0(t)] \\ [Q_{\text{int}}(t), H_{\text{int}}(t)] &= 0 \end{aligned} \tag{3.21}$$

assure the BRS invariance of the Hamilton operator (3.5).

Evaluating the commutators of the ghost charge operator Q_G, equation (2.48), with the various field operators, we find that the only nonvanishing results are given by

$$\begin{aligned} [Q_G, \boldsymbol{\omega}] &= i\boldsymbol{\omega}, \qquad [Q_G, \boldsymbol{\chi}] = -i\boldsymbol{\chi} \\ [Q_G, \boldsymbol{\Omega}] &= -i\boldsymbol{\Omega}, \qquad [Q_G, \mathbf{X}] = i\mathbf{X} \end{aligned} \tag{3.22}$$

As a consequence, the eigenvalues of the Hermitian operator Q_G turn out to be purely imaginary. This rather strange fact is related to the indefinite metric of the Fock space. We will comment on this in Section 3.3. Equations (3.22) imply that the fields $\boldsymbol{\omega}$ and $\mathbf{X}$ carry ghost number $N_G = -iQ_G = 1$, while $\boldsymbol{\chi}$ and $\boldsymbol{\Omega}$ carry ghost number $N_G = -iQ_G = -1$. In this notation, the BRS charge has ghost number $N_G = 1$,

$$\begin{aligned} [Q_G, Q_0(t)] &= iQ_0(t) \\ [Q_G, Q_{\text{int}}(t)] &= iQ_{\text{int}}(t) \end{aligned} \tag{3.23}$$

and the Hamilton operator obviously has $N_G = 0$,

$$[Q_G, H_0(t)] = [Q_G, H_{\text{int}}(t)] = 0 \tag{3.24}$$

due to the conservation of the ghost charge.

3.2. Interaction Picture

With the quantization, the fields have become operators in the Heisenberg picture that satisfy the Heisenberg equations of motion with the full Hamilton operator H in the commutator, i.e.,

$$i\frac{\partial}{\partial t} F(\mathbf{x}, t) = [F(\mathbf{x}, t), H] \tag{3.25}$$

The state vectors that define the Fock space in which the operators act are time-independent in the Heisenberg picture

$$i\frac{\partial}{\partial t}|\psi\rangle = 0 \tag{3.26}$$

It is useful to transform all the state vectors and the field operators into the interaction or Dirac picture using a unitary transformation $U(t)$ in the Fock space. This transformation satisfies the differential equation

$$i\frac{\partial}{\partial t} U(t) = U(t)H_{\text{int}}(t) \tag{3.27}$$

Thus, a Heisenberg state $|\psi\rangle$ transforms into a Dirac state $|\hat{\psi}(t)\rangle$ via

$$|\hat{\psi}(t)\rangle = U(t)|\psi\rangle \tag{3.28}$$

and a general Heisenberg operator $F(\mathbf{x}, t)$, which depends on $\mathbf{x}$ and t via the field operators, the spatial derivatives, and the canonical conjugate momenta, transforms into a general operator $\hat{F}(\mathbf{x}, t)$ in the interaction or Dirac picture according to the equation

$$\hat{F}(\mathbf{x}, t) = U(t)F(\mathbf{x}, t)U^{-1}(t) \tag{3.29}$$

Under this transformation all functional relations between field operators remain unchanged as long as they are expressed in terms of the field operators, the spatial derivatives, and the canonical momenta instead of the time derivatives. Thus, all canonical commutation rules (3.1)-(3.4) and also the Hamilton densities (2.38) and (2.39) remain invariant. However, the field equations and the relations between the canonical momenta and the time derivatives of the field operators are different in the Dirac picture, since we now have

$$i\frac{\partial}{\partial t} F(\mathbf{x}, t) = [\hat{F}(\mathbf{x}, t), \hat{H}_0] \tag{3.30}$$

for the time evolution of an operator and

$$i\frac{\partial}{\partial t} |\hat{\psi}(t)\rangle = \hat{H}_{\text{int}}(t)|\hat{\psi}(t)\rangle \tag{3.31}$$

i.e., the Schrödinger equation, for the time evolution of a state vector in the interaction picture. The defining relation for $U(t)$, equation (3.27), can be rewritten with the help of (3.29) as

$$i\frac{\partial}{\partial t} U(t) = \hat{H}_{\text{int}}(t)U(t) \tag{3.32}$$

Omitting the g-dependent terms in equations (3.6)-(3.15) according to (3.30), we can easily show that in the Dirac picture the field operators satisfy the noninteracting field equations, i.e.,

$$(i\gamma_\mu\partial^\mu - M)\hat{\psi} = \hat{\bar{\psi}}(i\gamma_\mu\overleftarrow{\partial}^\mu + M) = 0 \tag{3.33}$$

$$\Box\hat{\mathbf{A}}^\mu + (\lambda - 1)\partial^\mu\partial_\nu\hat{\mathbf{A}}^\nu = 0 \tag{3.34}$$

$$\Box\hat{\boldsymbol{\omega}} = \Box\hat{\boldsymbol{\chi}} = 0 \tag{3.35}$$

A transformation between solutions to the field equation (3.34) with different nonzero values of λ is obtained by setting

$$\hat{\mathbf{A}}^{\mu}_{\lambda_2} = \hat{\mathbf{A}}^{\mu}_{\lambda_1} + \left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right) \partial^{\mu} \hat{\boldsymbol{\xi}} \tag{3.36}$$

where $\hat{\boldsymbol{\xi}}$ is given by

$$\Box \hat{\boldsymbol{\xi}} = \lambda_1 \partial_{\mu} \hat{\mathbf{A}}^{\mu}_{\lambda_1} = \lambda_2 \partial_{\mu} \hat{\mathbf{A}}^{\mu}_{\lambda_2} \tag{3.37}$$

Here, $\hat{\mathbf{A}}^{\mu}_{\lambda_1}$ and $\hat{\mathbf{A}}^{\mu}_{\lambda_2}$ satisfy equation (3.34) with λ replaced by λ_1 and λ_2, respectively. Similarly, the field operators and the canonical conjugate momenta are related in the interaction picture by

$$\hat{\mathbf{\Pi}}^k = \partial^k \hat{\mathbf{A}}^0 - \partial^0 \hat{\mathbf{A}}^k, \qquad \hat{\mathbf{\Pi}}^0 = -\lambda \partial_{\mu} \hat{\mathbf{A}}^{\mu} \tag{3.38}$$

$$\hat{\mathbf{X}} = -i\partial_0 \hat{\boldsymbol{\omega}}, \qquad \hat{\mathbf{\Omega}} = i\partial_0 \hat{\boldsymbol{\chi}} \tag{3.39}$$

while the corresponding relations for the quark field operators, which do not involve time derivatives, remain unchanged and are thus given by equations (2.34). Based on equations (3.38) and (3.39), we easily arrive at the somewhat surprising relation

$$\hat{\mathcal{H}}_{\text{int}} = -\mathcal{L}_{\text{int}}(\hat{\phi}_i, \partial_{\mu}\hat{\phi}_i) + \tfrac{1}{2} g^2 (\hat{\mathbf{A}}^0 \times \hat{\mathbf{A}}^k) \cdot (\hat{\mathbf{A}}^0 \times \hat{\mathbf{A}}^k) \tag{3.40}$$

where the second term is due to the presence of derivative couplings in the Lagrange density (2.29). In this equation, $\hat{\mathcal{H}}_{\text{int}}$ and $\mathcal{L}_{\text{int}}(\hat{\phi}_i, \partial_{\mu}\hat{\phi}_i)$ are defined by equations (2.39) and (2.29), respectively, but now the arguments have been replaced by operators in the Dirac picture. Note that equation (2.40), which is the corresponding equation in the Heisenberg picture, differs from equation (3.40) in the sign of the second term.

We now introduce the so-called time-evolution operator $U(t, t_0)$, which is related to the unitary transformation $U(t)$ in (3.27) or (3.32) by

$$U(t, t_0) = U(t) U^{-1}(t_0) \tag{3.41}$$

The time-evolution operator $U(t, t_0)$ satisfies the same differential equation (3.32) as $U(t)$,

$$i \frac{\partial}{\partial t} U(t, t_0) = \hat{H}_{\text{int}}(t) U(t, t_0) \tag{3.42}$$

together with the initial condition

$$U(t_0, t_0) = \mathbb{1} \tag{3.43}$$

In contrast to the unitary transformation $U(t)$, the time evolution operator acts completely in the Dirac picture. Using the time independence of the Heisenberg state vectors (3.26) and equation (3.28), we arrive at

$$|\hat{\psi}(t)\rangle = U(t, t_0) |\hat{\psi}(t_0)\rangle \tag{3.44}$$

justifying the name "time-evolution operator" for $U(t, t_0)$. Of course, equations (3.44) and (3.42) are equivalent to the Schrödinger equation (3.31). The full quantum theory is now contained in the operator $U(t, t_0)$. The solution of the differential equation (3.42) with the initial condition (3.43) is given by Dyson's expansion in terms of n-dimensional integrals which involve time-ordered T-products of $\hat{H}^{\varepsilon}_{\text{int}}(t)$,

$$U^{\varepsilon}(t, t_0) = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int_{t_0}^{t} dt_1 \cdots \int_{t_0}^{t} dt_n \, T(\hat{H}^{\varepsilon}_{\text{int}}(t_1) \cdots \hat{H}^{\varepsilon}_{\text{int}}(t_n)) \tag{3.45}$$

Here we have introduced the usual adiabatic switching on of the interaction

$$\hat{H}^{\varepsilon}_{\text{int}}(t) = e^{-\varepsilon|t|} \hat{H}_{\text{int}}(t) \tag{3.46}$$

where ε is a small, positive quantity which makes it possible to study the limits of $U(t, t_0)$ for $t, t_0 \to \pm\infty$ as well.

We now want to determine the eigenstates and eigenvalues of the full Hamilton operator

$$\hat{H}(0) = \hat{H}_0 + \hat{H}_{\text{int}}(0) \tag{3.47}$$

Let $|\hat{\phi}_k\rangle$ be a complete and orthonormal set of eigenvectors of the noninteracing Hamiltonian $\hat{H}_0$ in the Dirac picture, $E_k^{(0)}$ being the eigenvalues, i.e.,

$$\hat{H}_0 |\hat{\phi}_k\rangle = E_k^{(0)} |\hat{\phi}_k\rangle \tag{3.48}$$

If the state vector given by

$$|\hat{\Psi}_k\rangle = \lim_{\varepsilon \to 0_+} \frac{U^{\varepsilon}(0, -\infty)|\hat{\Phi}_k\rangle}{\langle \hat{\Phi}_k | U^{\varepsilon}(0, -\infty) | \hat{\Phi}_k \rangle} \tag{3.49}$$

exists to all orders, then, due to the Gell-Mann and Low (1951) theorem, $|\hat{\Psi}_k\rangle$ is an eigenstate of the full Hamilton operator $\hat{H}(0)$ with the energy E_k, i.e.,

$$\hat{H}(0)|\hat{\Psi}_k\rangle = [\hat{H}_0 + \hat{H}_{\text{int}}(0)]|\hat{\Psi}_k\rangle = E_k |\hat{\Psi}_k\rangle \tag{3.50}$$

Multiplying this equation with $\langle \hat{\Phi}_k|$ from the left and using the Hermiticity property of $\hat{H}_0$, we immediately obtain

$$E_k - E_k^{(0)} = \frac{\langle \hat{\Phi}_k | \hat{H}_{\text{int}}(0) | \hat{\Psi}_k \rangle}{\langle \hat{\Phi}_k | \hat{\Psi}_k \rangle} \tag{3.51}$$

for the difference of the energy eigenvalues in the interacting and noninteracting systems, respectively. Moreover, introducing the eigenvectors (3.49), we easily arrive at

$$E_k - E_k^{(0)} = \lim_{\varepsilon \to 0_+} \frac{\langle \hat{\Phi}_k | \hat{H}_{\text{int}}(0) U^{\varepsilon}(0, -\infty) | \hat{\Phi}_k \rangle}{\langle \hat{\Phi}_k | U^{\varepsilon}(0, -\infty) | \hat{\Phi}_k \rangle} \tag{3.52}$$

Similarly, we can expand the eigenvectors of the interacting system in terms of the eigenstates of the noninteracting system, i.e.,

$$|\hat{\Psi}_k\rangle = \lim_{\varepsilon\to 0_+} \sum_{l=0}^{\infty} \frac{\langle\hat{\Phi}_l|U^{\varepsilon}(0,-\infty)|\hat{\Phi}_k\rangle}{\langle\hat{\Phi}_k|U^{\varepsilon}(0,-\infty)|\hat{\Phi}_k\rangle} |\hat{\Phi}_l\rangle \tag{3.53}$$

which also follows from equation (3.49). Finally, using Dyson's expansion (3.45), we can readily write down the energy shifts due to the interaction as

$$E_k - E_k^{(0)} = \lim_{\varepsilon\to 0_+} \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int_{-\infty}^{0} dt_1 \cdots \int_{-\infty}^{0} dt_n \times \langle\hat{\Phi}_k|T(\hat{H}^{\varepsilon}_{\text{int}}(0)\hat{H}^{\varepsilon}_{\text{int}}(t_1)\cdots\hat{H}^{\varepsilon}_{\text{int}}(t_n))|\hat{\Phi}_k\rangle_{\text{connected}} \tag{3.54}$$

where, for convenience, $\hat{H}^{\varepsilon}_{\text{int}}(0)$ has been placed inside the time-ordered T-product. The Wick decomposition of equation (3.54) will eventually lead to an expansion in terms of Feynman diagrams in coordinate space. If we restrict this sum to the so-called connected diagrams, the denominator that was present in equation (3.52) must be omitted, based on Goldstone's theorem.

3.3. Physical States

It is well known that covariant canonical quantization of gauge fields inevitably leads to a Fock space $\mathbb{V}$ with indefinite metric. This is most easily seen by realizing that, in a symbolic notation, the commutator

$$[A^{\mu}, \Pi^{\nu}] = ig^{\mu\nu} \tag{3.55}$$

results in

$$[c^{\mu}, c^{\nu+}] = -g^{\mu\nu} \tag{3.56}$$

for the creation and annihilation operators. The presence of negative norm states, such as

$$|1\rangle = c^{0+}|0\rangle; \qquad \langle 1|1\rangle = -\langle 0|0\rangle \tag{3.57}$$

in the Fock space is intimately related to the indefiniteness of the Minkowski metric of space-time $g^{\mu\nu}$, and endangers the probabilistic interpretation of the quantum theory (negative probabilities!). Thus, we can conclude that the whole indefinite metric Fock space $\mathbb{V}$ defined by the field operators of the theory is too large to describe the physical world. Further, the unconstrained presence of ghosts in the physical space would violate the usual spin-statistics connection, since the ghosts are spin-zero fermions. One of the main problems in covariant quantization is to define consistently a physical Hilbert space that does not contain the unphysical negative norm states.

In the Abelian quantum electrodynamics, the Gupta (1950)-Bleuler (1950) condition

$$\partial_\mu A^{\mu(+)}|\Psi_{\text{phys}}\rangle = 0 \tag{3.58}$$

is sufficient to guarantee a positive-definite subspace $\{|\Psi_{\text{phys}}\rangle\}$. Furthermore, since in this theory the four-divergence of the photon field $\partial_\mu A^\mu$ satisfies, even in the Heisenberg picture, the noninteracting d'Alembert equation, equation (3.58) can be shown to be consistent with time evolution. This method is not applicable to a non-Abelian gauge theory, e.g., quantum chromodynamics. In the Heisenberg picture, $\partial_\mu A_a^\mu$ is not a free field. In the interaction picture, where $\partial_\mu \hat{A}_a^\mu$ obeys the free d'Alembert equation by virtue of (3.34), the state vectors are time-dependent. Thus, an equation of the form (3.58) cannot be maintained for all times.

Kugo and Ojima (1979) developed a consistent formalism that generalizes the Gupta-Bleuler condition (3.58) to non-Abelian gauge theories, which we would like to adopt here. Using the time-independent BRS charge Q_B in (2.45) or (3.18), (3.19), we find that the physical subspace $\mathbb{V}_p$ consists of all state vectors satisfying the relation

$$Q_B|\Psi_{\text{phys}}\rangle = 0 \tag{3.59}$$

Based mainly on the algebra

$$Q_B^2 = 0, \qquad [Q_G, Q_B] = iQ_B \tag{3.60}$$

$\mathbb{V}_p$ can be shown to be positive semidefinite.

Of course, any state vector $|\Psi'\rangle$ that can be expressed as

$$|\Psi'\rangle = Q_B|\Psi\rangle \tag{3.61}$$

with an arbitrary $|\Psi\rangle$ trivially satisfies the physicality condition (3.59) due to the nilpotency of the BRS charge. However, it has zero norm and is orthogonal to $\mathbb{V}_p$. Thus, it cannot contribute to any measurable quantity. Moreover, the state $|\Psi'_{\text{phys}}\rangle$ that is obtained from the physical state $|\Psi_{\text{phys}}\rangle$ of (3.59) by applying the operator F on it, i.e.,

$$|\Psi'_{\text{phys}}\rangle = F|\Psi_{\text{phys}}\rangle \tag{3.62}$$

where the operator F either commutes or anticommutes with the BRS charge

$$[Q_B, F] = 0 \qquad \text{or} \qquad \{Q_B, F\} = 0 \tag{3.63}$$

is again a physical state. An operator satisfying one of equations (3.63) is called an observable.

It is interesting to note that the field equation (2.31) for the gluons can also be expressed in the form (Kugo and Ojima, 1979)

$$\partial_\nu \mathbf{F}^{\nu\mu} + g\mathbf{J}_C^\mu + \{Q_B, \mathscr{D}^\mu \boldsymbol{\chi}\} = 0 \tag{3.64}$$

where $\mathbf{J}_C^\mu$ is the color-carrying current (2.43). We deduce from this representation (3.64) that the gluon fields $\mathbf{A}^\mu$ obey, in the physical subspace $\mathbb{V}_p$ of (3.59), the generalized Maxwell equations

$$\langle \Psi'_{\text{phys}} | \partial_\nu \mathbf{F}^{\nu\mu} + g\mathbf{J}_C^\mu | \Psi_{\text{phys}} \rangle = 0 \tag{3.65}$$

Let us now discuss the condition (3.59) in the Dirac picture, especially with regard to the perturbative expansion (3.45)-(3.54). Using the relations

$$\begin{aligned} i\frac{\partial}{\partial t}\hat{Q}_0 &= 0 \\ [\hat{Q}_0, \hat{H}_{\text{int}}(t)] &= -i\frac{\partial}{\partial t}\hat{Q}_{\text{int}}(t) \\ [\hat{Q}_{\text{int}}(t), \hat{H}_{\text{int}}(t)] &= 0 \end{aligned} \tag{3.66}$$

that follow from (3.21), it can be shown that the BRS charge inherits the adiabatic damping factor $e^{-\varepsilon|t|}$ from the Hamiltonian

$$\hat{Q}_B^\varepsilon(t) = \hat{Q}_0 + e^{-\varepsilon|t|}\hat{Q}_{\text{int}}(t) \tag{3.67}$$

Since the noninteracting or asymptotic states $|\hat{\Phi}\rangle$, equation (3.48), correspond to the limit $t \to -\infty$, equation (3.59) translates into

$$\hat{Q}_0 |\hat{\Phi}_{\text{phys}}\rangle = 0 \tag{3.68}$$

Moreover, the algebra (3.60) guarantees that the state vector $|\hat{\Psi}_{\text{phys}}\rangle$ which develops adiabatically from $|\hat{\Phi}_{\text{phys}}\rangle$ in (3.68) according to (3.49) cannot destroy the positive semidefiniteness of $\mathbb{V}_p$.

Evaluating equation (3.18) in the Dirac picture, we obtain with the help of the field equation (3.34) and the definition of the conjugate momentum $\hat{\mathbf{\Pi}}^\mu$ in (3.38)

$$\hat{Q}_0 = \lambda \int d^3x \, (\partial^0 \hat{\boldsymbol{\omega}} \cdot \partial_\nu \hat{\mathbf{A}}^\nu - \hat{\boldsymbol{\omega}} \cdot \partial^0 \partial_\nu \hat{\mathbf{A}}^\nu) \tag{3.69}$$

This form of the asymptotic BRS charge reflects a close relation of the definition of the asymptotic physical subspace $\mathbb{V}_p$ of (3.68) with the Gupta-Bleuler condition (3.58).

4. CAVITY QUANTUM CHROMODYNAMICS

4.1. Boundary Conditions

In this section we want to find a consistent phenomenological description of the empirical fact that the color-carrying constituents of physical particles are confined in a finite region of space. This confinement is believed to be due to nonperturbative effects, which cannot be generated using the expansions in Section 3.2, equations (3.45)-(3.54). We therefore turn to a slightly different version of quantum chromodynamics in which the field operators are restricted to a domain in space V, which we call the cavity, its boundary being the closed surface ∂V.

Of course we want to make sure that this modified quantum chromodynamics resembles as closely as possible the original theory. In the cavity, the field operators will satisfy the field equations (2.30)-(2.33) in the Heisenberg picture, or (3.33)-(3.35) in the interaction picture. We will further demand that the conserved charges introduced in Section 2.4 remain conserved in the cavity. In order to discuss the consequences of the latter requirement (Chodos *et al.*, 1974a), let us describe the cavity V by its characteristic step function $\Theta(x)$,

$$\Theta(x) = \begin{cases} 1 & \text{for} \quad x \in V \\ 0 & \text{for} \quad x \notin V \end{cases} \tag{4.1}$$

which is related to the surface delta function $\delta(x)$ by

$$\partial_\mu \Theta(x) = n_\mu \delta(x) \tag{4.2}$$

Here, $n_\mu = (n_0, -\mathbf{n})$ denotes a spacelike unit vector

$$n_\mu n^\mu = -1 \tag{4.3}$$

with $\mathbf{n}$ perpendicular to the surface ∂V and pointing outward.

A conserved charge Q is given by the space integral over the zero component of a conserved current

$$Q = \int_V d^3x \, J^0 = \int_{\mathbb{R}^3} d^3x \, \Theta J^0 \tag{4.4}$$

Using equation (4.2) and the conservation of the current $\partial_\mu J^\mu = 0$, we easily find the time derivative to be

$$\frac{\partial}{\partial t} Q = \int_{\partial V} d\Omega \, n_\mu J^\mu \tag{4.5}$$

where $d\Omega$ is the two-dimensional surface element. Hence, if the conserved current J^μ satisfies the boundary condition

$$n_\mu J^\mu(x) = 0, \qquad x \in \partial V \tag{4.6}$$

the charge (4.4) will be time independent in the cavity.

Inspecting the currents given in Section 2.4, it is readily verified that equation (4.6) holds if we impose the following set of boundary conditions on the field operators in the Heisenberg picture:

$$(in_\mu\gamma^\mu - 1)\psi|_{\partial V} = \bar{\psi}(in_\mu\gamma^\mu + 1)|_{\partial V} = 0 \tag{4.7}$$

$$n_\mu \mathbf{F}^{\mu\nu}|_{\partial V} = n_\mu \mathbf{A}^\mu|_{\partial V} = n_\mu \partial^\mu(\partial_\nu \mathbf{A}^\nu)|_{\partial V} = 0 \tag{4.8}$$

$$n_\mu \partial^\mu \boldsymbol{\omega}|_{\partial V} = n_\mu \partial^\mu \boldsymbol{\chi}|_{\partial V} = 0 \tag{4.9}$$

Equations (4.7)-(4.9) are essentially those introduced by the MIT group (Chodos *et al.*, 1974a; Hansson and Jaffe, 1983; Goldhaber *et al.*, 1983, 1986). Of course, one could think of more complicated boundary conditions than the above. However, the set (4.7)-(4.9) has the advantage of being linear in the field operators and independent of the strong coupling constant g.

The first of equations (4.8) implies that the space integral of $\partial_\mu \mathbf{F}^{\mu 0}$ over the cavity volume vanishes. Combining this result with the field equation for gluons in the form (3.64), we can express the color generator $\mathbf{Q}_C$ as

$$g\mathbf{Q}_C = g\int_V d^3x\, \mathbf{J}_C^0 = \int_V d^3x\, [\partial_\mu \mathbf{F}^{\mu 0} + \{Q_B, \mathscr{D}^0\boldsymbol{\chi}\}] = -\left\{Q_B, \int_V d^3x\, \mathscr{D}^0\boldsymbol{\chi}\right\} \tag{4.10}$$

This representation leads immediately to the conclusion that

$$\langle\Psi'_{\text{phys}}|\mathbf{Q}_C|\Psi_{\text{phys}}\rangle = 0 \tag{4.11}$$

i.e., the color charge vanishes in the subspace of physical states in the cavity.

Localizing the field operators in a cavity, we are violating translational invariance of the theory. Consequently, the 3-momentum will not be conserved. If we restrict ourselves to a static cavity ($n^0 = 0$), the Hamiltonian will be time-independent and the energy will be conserved. In this static case with a time-independent surface ∂V, the boundary conditions (4.7)-(4.9), which are formulated in the Heisenberg picture, translate easily into the Dirac picture as follows:

$$(in_k\gamma^k - 1)\hat{\psi}|_{\partial V} = \hat{\bar{\psi}}(in_k\gamma^k + 1)|_{\partial V} = 0 \tag{4.12}$$

$$n_k(\partial^k \hat{\mathbf{A}}^\nu - \partial^\nu \hat{\mathbf{A}}^k)|_{\partial V} = n_k \hat{\mathbf{A}}^k|_{\partial V} = n_k \partial^k(\partial_\nu \hat{\mathbf{A}}^\nu)|_{\partial V} = 0 \tag{4.13}$$

$$n_k \partial^k \hat{\boldsymbol{\omega}}|_{\partial V} = n_k \partial^k \hat{\boldsymbol{\chi}}|_{\partial V} = 0 \tag{4.14}$$

4.2. Expansions in Cavity Modes

The field operators can be expanded in terms of a complete set of field functions or cavity modes that satisfy the same field equations and boundary conditions as the field operators in the Dirac picture. (For simplicity, we use here the so-called Feynman gauge with $\lambda = 1$.) The operator character of the field operators is now carried by the expansion coefficients. In the static case the time dependence of the quark, gluon, and ghost field functions can be factored out, yielding time-independent field equations and boundary conditions for the quark, gluon, and ghost field functions. In the simplest case of a spherically symmetric and static cavity, we arrive at the cavity modes that are discussed in Appendix A.

We now expand the quark field operator in terms of the cavity modes, splitting this operator into positive and negative frequency parts,

$$\begin{aligned}\hat{\psi}_{cf}(x) &= \hat{\psi}^{(+)}_{cf}(x) + \hat{\psi}^{(-)}_{cf}(x)\\ &= \sum_{\substack{\kappa\mu\\ \nu>0}} [\hat{a}_{cn} u_n(\mathbf{x})\, e^{-i\varepsilon_n t} + \hat{b}^+_{cn} u_{-n}(\mathbf{x})\, e^{i\varepsilon_n t}]. \end{aligned} \tag{4.15}$$

Here we have made use of the symmetry relation

$$\varepsilon_{-n} = -\varepsilon_n \tag{4.16}$$

where n and $-n$ denote sets of quantum numbers defined by

$$n = \{f, \nu, \kappa, (\mu)\}, \qquad -n = \{f, -\nu, -\kappa, (-\mu)\} \tag{4.17}$$

The sum in equation (4.15) extends over all Dirac and magnetic quantum numbers κ and μ, respectively, and the positive radial quantum numbers ν. Similarly, we have for the Hermitian adjoint quark field operators

$$\begin{aligned}\hat{\bar{\psi}}_{cf}(x) &= \hat{\bar{\psi}}^{(+)}_{cf}(x) + \hat{\bar{\psi}}^{(-)}_{cf}(x)\\ &= \sum_{\substack{\kappa\mu\\ \nu>0}} [\hat{a}^+_{cn} \bar{u}_n(\mathbf{x})\, e^{i\varepsilon_n t} + \hat{b}_{cn} \bar{u}_{-n}(\mathbf{x})\, e^{-i\varepsilon_n t}] \end{aligned} \tag{4.18}$$

The operators $\hat{a}^+_{cn}$ ($\hat{a}_{cn}$) and $\hat{b}^+_{cn}$ ($\hat{b}_{cn}$) are defined for $\nu > 0$ and describe the creation (annihilation) of quarks and antiquarks, respectively. Indeed, using the anticommutation relations (3.1) and the orthonormality condition (A16), we immediately arrive at the anticommutation relations for the quarks and antiquarks:

$$\{\hat{a}_{cn}, \hat{a}^+_{c'n'}\} = \{\hat{b}_{cn}, \hat{b}^+_{c'n'}\} = \delta_{cc'}\delta_{nn'} \tag{4.19}$$

Similarly, the gluon field operators can be expanded in terms of the cavity modes defined in Appendix A, yielding

$$\begin{aligned}\hat{A}^\mu_a(x) &= \hat{A}^{\mu(+)}_a(x) + \hat{A}^{\mu(-)}_a(x)\\ &= \sum_{\substack{\Sigma JM\\ N>0}} (2\Omega^\Sigma_m)^{-1/2}[\hat{c}^\Sigma_{am} a^{\mu\Sigma}_m(\mathbf{x}) \exp(-i\Omega^\Sigma_m t) + \hat{c}^{\Sigma+}_{am} a^{\mu\Sigma}_m(\mathbf{x})^* \exp(i\Omega^\Sigma_m t] \end{aligned} \tag{4.20}$$

Here, $\Sigma = 0, \mathscr{L}, \mathscr{M}, \mathscr{E}$ denote the scalar, longitudinal, transverse magnetic, and transverse electric polarizations of the gluon cavity modes, respectively. The symbol m stands for a set of quantum numbers given by

$$m = \{N, J, (M)\} \tag{4.21}$$

where J and M are the angular momentum quantum numbers and N denotes the radial quantum number. The operators $\hat{c}_{am}^{\Sigma +}$ and $\hat{c}_{am}^{\Sigma}$ can be interpreted as creation and annihilation operators for a gluon with polarization Σ. Indeed, based on equations (3.2), (A34), and (A35), we arrive at the commutation relations

$$[\hat{c}_{am}^{\Sigma}, \hat{c}_{a'm'}^{\Sigma' +}] = -g^{\Sigma\Sigma'}\delta_{aa'}\delta_{mm'} \tag{4.22}$$

where the metric $g^{\Sigma\Sigma'}$ is given by equation (A37).

Finally, the ghost field operators can be expanded in terms of cavity modes as well, yielding

$$\begin{aligned}\hat{\omega}_a(x) &= \hat{\omega}_a^{(+)}(x) + \hat{\omega}_a^{(-)}(x)\\ &= \sum_{\substack{JM\\ N>0}} (2\Omega_m^0)^{-1/2}[\hat{d}_{am}a_m^0(\mathbf{x})\exp(-i\Omega_m^0 t) + \hat{d}_{am}^{+}a_m^0(\mathbf{x})^* \exp(i\Omega_m^0 t)]\end{aligned} \tag{4.23}$$

and

$$\begin{aligned}\hat{\chi}_a(x) &= \hat{\chi}_a^{(+)}(x) + \hat{\chi}_a^{(-)}(x)\\ &= \sum_{\substack{JM\\ N>0}} (2\Omega_m^0)^{-1/2}[\hat{e}_{am}a_m^0(\mathbf{x})\exp(-i\Omega_m^0 t) + \hat{e}_{am}^{+}a_m^0(\mathbf{x})^* \exp(i\Omega_m^0 t)]\end{aligned} \tag{4.24}$$

Using equations (3.3), (3.4), and (A35), we can easily show that the only nonvanishing anticommutators are

$$\{\hat{d}_{am}, \hat{e}_{a'm'}^{+}\} = -\{\hat{d}_{am}^{+}, \hat{e}_{a'm'}\} = i\delta_{aa'}\delta_{mm'} \tag{4.25}$$

The zero-particle state or the asymptotic vacuum $|\hat{0}\rangle$ is the state with norm one that is annihilated by all the quark, antiquark, gluon, and ghost annihilation operators

$$\begin{aligned}\hat{a}_{cn}|\hat{0}\rangle &= \hat{b}_{cn}|\hat{0}\rangle = 0\\ \hat{c}_{am}^{\Sigma}|\hat{0}\rangle &= \hat{d}_{am}|\hat{0}\rangle = \hat{e}_{am}|\hat{0}\rangle = 0\\ \langle\hat{0}|\hat{0}\rangle &= 1\end{aligned} \tag{4.26}$$

The Fock space of asymptotic states, which is, by assumption, complete and thus contains all the states of the interacting theory, is obtained in the standard way by applying any combination of creation operators on the zero-particle state (4.26).

Based on the expansions (4.20), (4.23), and (4.24) and the orthogonality of the cavity modes (A35), the g-independent part Q_0 of the BRS charge, equation (3.69), is readily found to be

$$\hat{Q}_0 = -\sum_{am} \Omega_m^0[(\hat{c}_{am}^{\mathscr{L}} - \hat{c}_{am}^0)^{+}\hat{d}_{am} + \hat{d}_{am}^{+}(\hat{c}_{am}^{\mathscr{L}} - \hat{c}_{am}^0)] \tag{4.27}$$

Thus, sufficient conditions that an asymptotic state $|\hat{\Phi}\rangle$ satisfies the physicality criterion (3.68) can be stated as

$$(\hat{c}^{\mathscr{L}}_{am} - \hat{c}^{0}_{am})|\hat{\Phi}_{\text{phys}}\rangle = 0; \qquad \hat{d}_{am}|\hat{\Phi}_{\text{phys}}\rangle = 0 \tag{4.28}$$

Here, we see the close relation of equation (3.68) with the Gupta–Bleuler condition (3.58). Of course, the asymptotic vacuum $|\hat{0}\rangle$ is a physical state due to equation (4.26).

Similarly, we can write down the normal-ordered, noninteracting part of the Hamiltonian (2.38) in the Dirac picture as

$$:\hat{H}_0: = \sum_{cn} \varepsilon_n(\hat{a}^+_{cn}\hat{a}_{cn} + \hat{b}^+_{cn}\hat{b}_{cn}) + \sum_{\substack{am \\ \Sigma = \mathscr{M}, \mathscr{E}}} \Omega^{\Sigma}_m \hat{c}^{\Sigma+}_{am}\hat{c}^{\Sigma}_{am} + \{\hat{Q}_0, \hat{K}\} \tag{4.29}$$

where the fermionic operator $\hat{K}$ is given by

$$\hat{K} = \frac{i}{2} \sum_{am} [\hat{e}^+_{am}(\hat{c}^{\mathscr{L}}_{am} + \hat{c}^{0}_{am}) - (\hat{c}^{\mathscr{L}}_{am} + \hat{c}^{0}_{am})^+ \hat{e}_{am}] \tag{4.30}$$

The last term in equation (4.28), which represents the contribution to $\hat{H}_0$ from the unphysical longitudinal and scalar gluon fields and the ghost fields, has been cast into a form that shows explicitly that its matrix elements, taken between the physical states (3.68), vanish.

5. TWO-PARTICLE INTERACTIONS

5.1. Introduction

We now turn to the discussion of the various types of interactions that arise between quarks, antiquarks and gluons in second-order perturbation theory. Truncating the perturbation expansion (3.54) after the first two terms, we obtain the energy shifts of second order in the strong coupling constant g

$$\begin{aligned} E_k - E^{(0)}_k &= \langle\hat{\Phi}_k|\hat{H}_{\text{int}}(0)|\hat{\Phi}_k\rangle \\ &\quad - i \lim_{\varepsilon\to 0_+} \int_{-\infty}^{0} dt \langle\hat{\Phi}_k|T(\hat{H}_{\text{int}}(0)\hat{H}^{\varepsilon}_{\text{int}}(t))|\hat{\Phi}_k\rangle_{\text{connected}} \end{aligned} \tag{5.1}$$

Here $|\hat{\Phi}_k\rangle$ denotes an eigenvector of the noninteracting Hamiltonian $:\hat{H}_0:$, equation (4.29), with the eigenvalue $E^{(0)}_k$. Instead of $\hat{H}_0$, we make use of the normal-ordered operator $:\hat{H}_0:$, which differs from $\hat{H}_0$ by the (infinite) vacuum expectation value of $\hat{H}_0$ and thus does not contribute to the energy shift. $\hat{H}^{\varepsilon}_{\text{int}}(t)$ is the interaction Hamiltonian, equation (2.39), with the adiabatic damping factor attached. Due to the presence of the four-gluon vertex, the normal-ordered interaction Hamiltonian $:\hat{H}_{\text{int}}(t):$ differs from

$\hat{H}_{\text{int}}(t)$ by a nontrivial operator involving the product of the gluon fields $\mathbf{A}^k \cdot \mathbf{A}^l$. In contrast to quantum chromodynamics in free space, this operator is not diagonal in the space indices k and l, due to the violation of translation and Lorentz invariance in the cavity. Thus, this term cannot be absorbed into a mass renormalization of the gluons and must be kept in the interaction Hamiltonian. It is, of course, possible to subtract from $\hat{H}^{\varepsilon}_{\text{int}}(t)$ its (infinite) vacuum expectation value. Hereby the phase of the time-evolution operator $U^{\varepsilon}(t, t_0)$ is changed, but the form of the states (3.49) is not affected by this subtraction procedure.

In this paper we restrict ourselves to the study of nondivergent tree diagrams, shown in Figure 2, in order to avoid the problems related to the renormalization of Feynman diagrams in the cavity. A first promising step in solving these renormalization problems in "bagged" quantum chromodynamics has recently been made with the development of the multiple reflection formalism (Hansson and Jaffe, 1983; Goldhaber *et al.*, 1983, 1986).

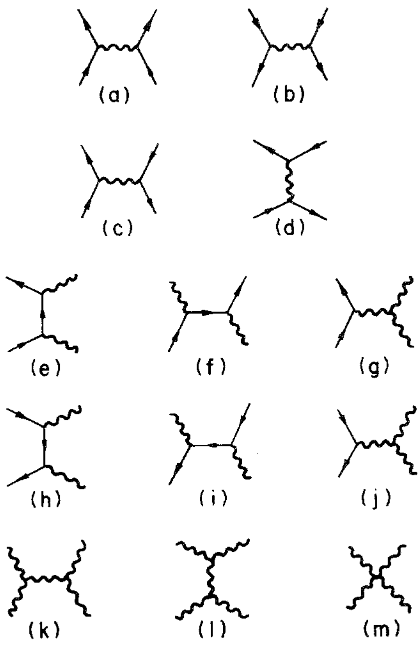


Fig. 2. The Feynman diagrams representing the second-order interactions discussed in this work: (a) the quark-quark and (b) antiquark-antiquark interactions through gluon exchange; the quark-antiquark interactions through (c) gluon exchange and (d) annihilation; the quark- or antiquark-gluon interactions through (e, h) the direct and (f, i) the exchange Compton diagrams and (g, j) the gluon exchange; the gluon-gluon interaction through (k) the gluon exchange, (l) the annihilation, and (m) the elementary four-gluon vertex.

The state vector $|\hat{\Phi}_k\rangle$ in equation (5.1) consists, in general, of quarks, antiquarks, gluons, and ghosts. Here, we assume that $|\hat{\Phi}_k\rangle$ is a physical state with ghost number zero and positive norm

$$\hat{Q}_0|\hat{\Phi}_k\rangle = 0; \qquad \hat{Q}_G|\hat{\Phi}_k\rangle = 0; \qquad \langle\hat{\Phi}_k|\hat{\Phi}_k\rangle = 1 \tag{5.2}$$

Note that the Gell-Mann and Low theorem must be slightly modified for states with nonvanishing ghost number, since the norm of such a state is zero, and the denominator in equations (3.49) and (3.15)-(3.53) vanishes. In order to satisfy all requirements of equation (5.2), we take the asymptotic state $|\hat{\Phi}_k\rangle$ to contain only quarks, antiquarks, and the two physical degrees of freedom of the gluon: the transverse electric and magnetic polarization modes.

In general, $E_k^{(0)}$ is degenerate and therefore several orthogonal eigenvectors $|\hat{\Phi}_k\rangle, |\hat{\Phi}_{k'}\rangle, \ldots$ belong to this eigenvalue. A linear combination of these vectors is again an eigenvector of $:\hat{H}_0:$ with the same eigenvalue and can be used in the Gell-Mann and Low formula (3.49). We are thus led to consider the off-diagonal matrix elements of the right-hand side of (5.1) as well,

$$V_{k'k} = \langle\hat{\Phi}_{k'}|\hat{H}_{\text{int}}(0)|\hat{\Phi}_k\rangle - i \lim_{\varepsilon\to 0_+} \int_{-\infty}^{0} dt\, \langle\hat{\Phi}_{k'}|T(\hat{H}_{\text{int}}(0)\hat{H}^{\varepsilon}_{\text{int}}(t))|\hat{\Phi}_k\rangle_{\text{connected}} \tag{5.3}$$

The energy shifts and the corresponding eigenstates are then obtained by diagonalizing the matrix $V_{k'k}$.

5.2. Quark–Quark Interaction

As an example, let us now, for a two-quark system, calculate the energy shift due to the one-gluon-exchange interaction. The eigenstates of $:\hat{H}_0:$ are given by

$$|\hat{\Phi}_k\rangle = \hat{a}^+_{c_1 n_1}\hat{a}^+_{c_2 n_2}|\hat{0}\rangle \tag{5.4}$$

The part of $H_{\text{int}}(t)$ that describes the two-quark, one-gluon vertex can be obtained from the first term in the Hamilton density (2.39) integrating over the volume

$$\hat{H}_{\text{int}}(t) = -g \int d^3x\, \hat{\bar{\psi}}(x)\gamma_\mu(\lambda_a/2)\hat{\psi}(x)\hat{A}^\mu_a(x) + \text{other terms} \tag{5.5}$$

Inserting this operator into equation (5.3) and using Wick's theorem to expand the time-ordered into normal-ordered products, we arrive at the matrix element

$$V_{k'k} = -ig^2(\gamma_\mu)_{\alpha'\alpha}(\gamma_\nu)_{\beta'\beta}(\lambda_a/2)_{c'c}(\lambda_b/2)_{d'd}\delta_{f'f}\delta_{g'g}$$
$$\times \lim_{\varepsilon\to 0_+}\int_{-\infty}^{0} dt\, e^{-\varepsilon|t|}\int d^3x \int d^3y\, \langle\hat{0}|T(\hat{A}_a^\mu(x)\hat{A}_b^\nu(y))|\hat{0}\rangle$$
$$\times\langle\hat{\Phi}_{k'}|:\hat{\bar{\psi}}^{(+)}_{c'f'\alpha'}(x)\hat{\psi}^{(+)}_{cf\alpha}(x)\hat{\bar{\psi}}^{(+)}_{d'g'\beta'}(y)\hat{\psi}^{(+)}_{dg\beta}(y):|\hat{\Phi}_k\rangle \tag{5.6}$$

Here we have picked out the term where the gluon fields are contracted and introduced the coordinates $x = (\mathbf{x}, 0)$ and $y = (\mathbf{y}, t)$. As usual, a summation over all repeated indices is understood. Expanding the quark field operators into cavity modes, as given in equations (4.15) and (4.18), and using the explicit form of the gluon propagator (B7), we arrive at the matrix element

$$V_{k'k} = -g^2\left(\frac{\lambda_a}{2}\right)_{c'c}\left(\frac{\lambda_a}{2}\right)_{d'd}\delta_{f'f}\delta_{g'g}\frac{g^{\Sigma\Sigma}}{2\Omega_m^\Sigma}\frac{1}{\Omega_m^\Sigma+\varepsilon_{p'}-\varepsilon_p}$$
$$\times\int d^3x\,\bar{u}_{n'}(\mathbf{x})\gamma_\mu u_n(\mathbf{x})a_m^{\mu\Sigma}(\mathbf{x})\int d^3y\,\bar{u}_{p'}(\mathbf{y})\gamma_\nu u_p(\mathbf{y})a_m^{\nu\Sigma}(\mathbf{y})^*$$
$$\times\langle\hat{\Phi}_{k'}|\hat{a}^+_{c'n'}\hat{a}^+_{d'p'}\hat{a}_{cn}\hat{a}_{dp}|\hat{\Phi}_k\rangle \tag{5.7}$$

where the energy denominator arises from the time integration. Thus, $V_{k'k}$ can be interpreted as the matrix element of the two-body operator

$$\hat{V} = g^2\left(\frac{\lambda_a}{2}\right)_{c'c}\left(\frac{\lambda_a}{2}\right)_{d'd}\delta_{f'f}\delta_{g'g}\frac{g^{\Sigma\Sigma}}{2\Omega_m^\Sigma}\frac{Q^\Sigma_{n'nm}\tilde{Q}^\Sigma_{p'pm}}{\Omega_m^\Sigma+\varepsilon_{p'}-\varepsilon_p}\hat{a}^+_{c'n'}\hat{a}^+_{c'p'}\hat{a}_{cn}\hat{a}_{dp} \tag{5.8}$$

which describes the one-gluon-exchange interaction between two quarks. Here we have made use of the definition of the quark–gluon vertex functions (C1) and (C1a).

In order to describe a complex many-quark system, it is convenient to have the two-body operator (5.8) expressed in first instead of second quantization. The Fock space can be embedded in the space given by the direct-product wave functions of the quarks. The Pauli principle is taken care of by restricting the Hilbert space to the subspace defined by the antisymmetrized product wave functions of the quarks. The two-body operator V_{12} corresponding to $\hat{V}$ and acting on the quantum numbers of the first and second quark can be written as

$$V_{12} = \frac{g^2}{4\pi R}\mathbf{F}_1\cdot\mathbf{F}_2\sum_J \mu_{12}(J)K_{12}(J) \tag{5.9}$$

Here J describes the angular momentum exchanged between the quarks and $\mathbf{F}_i(i = 1, 2)$ denotes the color generator in the fundamental representation. The operators $\mu_{12}(J)$ and $K_{12}(J)$ are defined as two-body operators that act on the radial and angular part of the two-body wave function, respectively. These are readily defined in terms of their matrix elements. Using equations (C6) and (C7) to decompose the vertex functions $Q^{\Sigma}_{nn'm}$ in equation (5.8) into radial and angular parts, we arrive at

$$\langle n_1', n_2'|K_{12}(J)|n_1, n_2\rangle = (2J+1)\sum_M (-1)^M F_{JM}(n_1', n_1)F_{J-M}(n_2', n_2) \quad (5.10)$$

Here $|n_1, n_2\rangle$ denotes the direct product of the Dirac spinors (A2), and the factor $F_{JM}(n', n)$ arises from the angular integration (C8)

$$F_{JM}(n', n) = (-1)^{\mu'+1/2}\hat{j}'\hat{J}\hat{j}\begin{pmatrix} j' & J & j \\ \frac{1}{2} & 0 & -\frac{1}{2}\end{pmatrix}\begin{pmatrix} j' & J & j \\ -\mu' & M & \mu\end{pmatrix} \quad (5.11)$$

Similarly, we can define

$$\langle n_1', n_2'|\mu_{12}(J)|n_1, n_2\rangle = \sum_{\substack{N>0 \\ \Sigma}} \frac{g^{\Sigma\Sigma}\eta_\Sigma}{2(2J+1)} S^{\Sigma}_{n_1'n_1m} S^{\Sigma}_{n_2'n_2m}$$

$$\times \frac{1}{R^2\Omega^{\Sigma}_m}\left[\frac{1}{\Omega^{\Sigma}_m + \varepsilon_{n_1'} - \varepsilon_{n_1}} + \frac{1}{\Omega^{\Sigma}_m + \varepsilon_{n_2'} - \varepsilon_{n_2}}\right] \quad (5.12)$$

where the matrix element $S^{\Sigma}_{nn'm}$ is related to the radial integrals $R^{\Sigma}_{nn'm}$ (Viollier *et al.*, 1971, 1983) defined in equations (C9)-(C12),

$$S^{\Sigma}_{nn'm} = R^{\Sigma}_{nn'm}\frac{1 - g^{\Sigma\Sigma}\eta_\Sigma(-1)^{l+J+l'}}{2} \quad (5.13)$$

The second factor in equation (5.13) governs the parity selection rule, and $g^{\Sigma\Sigma}$ and η_Σ are defined in (A37) and (A43), respectively.

Let us now consider the case where the quarks in the initial and final states carry total angular momentum $j = \frac{1}{2}$, which means that the Dirac quantum number κ takes the values $+1$ or -1. Using the Wigner-Eckart theorem, one can easily show that the only nonvanishing $K_{12}(J)$ are

$$K_{12}(0) = \mathbb{1}_1\mathbb{1}_2 \qquad \text{and} \qquad K_{12}(1) = 4\mathbf{S}_1\cdot\mathbf{S}_2 \quad (5.14)$$

where $\mathbb{1}_i$ and $\mathbf{S}_i$ denote the unit and the spin operator, respectively, acting on the ith quark. Thus, for quarks in the angular momentum state $j = \frac{1}{2}$, we arrive at the well-known expression (DeGrand *et al.*, 1975; Johnson, 1975; Viollier *et al.*, 1983)

$$V_{12} = \frac{g^2}{4\pi R}\mathbf{F}_1\cdot\mathbf{F}_2[\mu_{12}(0) + 4\mu_{12}(1)\mathbf{S}_1\cdot\mathbf{S}_2] \quad (5.15)$$

which is very convenient for the calculation of the properties of many-quark systems.

In a similar way we can determine the two-body operators describing the interactions corresponding to all other nondivergent Feynman graphs of order g^2 that do not involve the ghosts. The resulting two-body operators in first or second quantization are presented in Appendix D.

5.3. Energy Shifts for Low-Lying Cavity Modes

We now discuss the various two-body interactions between the quarks, antiquarks, and gluons. For simplicity, we consider here massless up and down quarks and antiquarks and gluons occupying the lowest cavity modes. The corresponding quantum numbers are $(p, q)J^{\pi}I = (1, 0)\frac{1}{2}^{+}\frac{1}{2}$ for the quarks and $(p, q)J^{\pi}I = (1, 1)1^{+}0$ for the gluons. Here, the two integers (p, q) characterize the irreducible representation of $SU(3)_{\text{color}}$, the dimensionality of the representation and the eigenvalue of the quadratic Casimir operator being given by

$$SU(3){:}\quad \begin{aligned} N(p, q) &= \tfrac{1}{2}(p+1)(q+1)(p+q+2) \\ C(p, q) &= \tfrac{1}{3}(p^2+pq+q^2)+p+q \end{aligned} \tag{5.16}$$

Similarly, the irreducible representations of $SU(2)_{\text{spin}}$ and $SU(2)_{\text{isospin}}$ are described by the spin J and the isospin I, respectively, and the dimensionality and eigenvalue of the Casimir operator are given in this representation by

$$SU(2){:}\quad \begin{aligned} N(J) &= 2J+1 \\ C(J) &= J(J+1) \end{aligned} \tag{5.17}$$

Finally, the quantity π stands for the parity of the state.

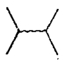
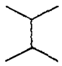
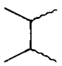
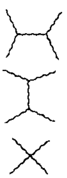
The two particles will in general be embedded into a larger many-body system. This system must be in a singlet representation of the color group $SU(3)_{\text{color}}$, since, according to equation (4.11), physical states must be colorless. This restriction does not apply to a subset of the constituents of the many-body system and therefore two particles of a larger system can be coupled to any of the allowed quantum numbers. Of course, identical particles must obey the Fermi–Dirac statistics for fermions and the Bose–Einstein statistics for bosons.

In Table I we have collected or all diagrams of Figure 2 the corresponding dimensionless interaction operators Δ_{12}, which are defined by

$$\Delta_{12} = \frac{4\pi R}{g^2} V_{12} \tag{5.18}$$

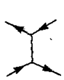
Tables II–IV contain the matrix elements of the two-body operators μ_{12} and ρ_{12}.

Table I. The Dimensionless Two-Body Interaction Operators Δ_{12} for the Various Diagrams of Figure 2[a].

Diagram	$\Delta_{12}=(4\pi R/g^2)V_{12}$
	$F_{12}[\mu_{12}(0)+4\mu_{12}(1)S_{12}]$
	$(\frac{1}{4}-T_{12})(F_{12}+\frac{4}{3})[\mu_{12}(0)(\frac{1}{4}-S_{12})+\mu_{12}(1)(\frac{3}{4}+S_{12})]$
	$\frac{1}{18}(4F_{12}^2-1)[\mu_{12}(1)(\frac{1}{2}-S_{12})+\mu_{12}(2)(1+S_{12})]$
	$\frac{1}{18}(4F_{12}^2+6F_{12}-1)[\mu_{12}(1)(\frac{1}{2}+S_{12})+\mu_{12}(2)(1-S_{12})]$
	$F_{12}[\mu_{12}(0)+\mu_{12}(1)S_{12}]$
	$F_{12}[\rho_{12}(0)+\rho_{12}(1)S_{12}+\rho_{12}(2)S_{12}^2]$

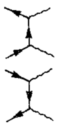
[a]We use the abbreviations $F_{12}=\mathbf{F}_1\cdot\mathbf{F}_2$, $S_{12}=\mathbf{S}_1\cdot\mathbf{S}_2$, and $T_{12}=\mathbf{T}_1\cdot\mathbf{T}_2$ for the product of color, spin, and isospin generators, respectively. The matrix elements of the operators $\mu_{12}(J)$ and $\rho_{12}(k)$ are given in Tables II–IV.

Table II. Matrix Elements of the Operators $\mu_{12}(J)$ for the Interactions via Gluon Exchange between Two Quarks, Two Antiquarks, or a Quark-Antiquark Pair and via Annihilation into a Gluon for the Quark-Antiquark System.[a]

Diagram	(κ_1, κ_2)	$\mu_{12}(0)$	$\mu_{12}(1)$
	$(-,-)$	0.0098 (0)	−0.1770 (−0.1770)
	$(-,+)_+$	0.0353 (0)	−0.1432 (−0.2082)
	$(-,+)_-$	0.0352 (0)	−0.0796 (−0.0146)
	$(+,+)$	0.1321 (0)	−0.1173 (−0.1173)
	$(-,-)$	0 (0)	0.1875 (−0.0124)
	$(-,+)_+$	0 (0)	0.0494 (0.0494)
	$(-,+)_-$	0.1806 (0)	0 (0)
	$(+,+)$	0 (0)	0.1055 (0.0135)

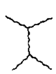
[a]The values are given for massless quarks in the lowest energy cavity mode. The numbers in parentheses are the contributions from the transverse polarizations of the virtual gluon. The mixed states correspond to $(-,+)_\pm=(1/\sqrt{2})[(-,+)\pm(+,-)]$, and $\kappa_1, \kappa_2=\pm 1$ denote the Dirac quantum numbers of the modes.

Table III. Matrix Elements of the Operators $\mu_{12}(J)$ for the Various Diagrams of the Quark (Antiquark)-Gluon Interaction[a].

Diagram	(κ, Σ)	$\mu_{12}(1)$	$\mu_{12}(2)$
	$(-, \mathcal{M})$	0.5616	0.0632
	$(-, \mathcal{E})$	0.4240	0.0346
	$(+, \mathcal{M})$	0.3175	0.0411
	$(+, \mathcal{E})$	0.4915	0.0286
	$(-, \mathcal{M})$	−0.3298	0.2101
	$(-, \mathcal{E})$	−0.0116	0.0913
	$(+, \mathcal{M})$	0.1130	0.0673
	$(+, \mathcal{E})$	−0.3076	0.0617
		$\mu_{12}(0)$	$\mu_{12}(1)$
	$(-, \mathcal{M})$	−0.0053 (0)	−0.4900 (−0.4900)
	$(-, \mathcal{E})$	0.0336 (0)	−0.4009 (−0.4009)
	$(+, \mathcal{M})$	−0.0214 (0)	−0.3204 (−0.3204)
	$(+, \mathcal{E})$	0.1347 (0)	−0.3900 (−0.3900)

[a]The numbers in parentheses represent the contributions to the gluon exchange from the transverse polarizations of the virtual gluon.

Table IV. Matrix Elements of the $\rho_{12}(k)$ Operators for the Gluon-Exchange, the Annihilation, and the Four-Gluon Vertex Interactions in a Two-Gluon System[a].

Diagram	(Σ_1, Σ_2)	$\rho_{12}(0)$	$\rho_{12}(1)$	$\rho_{12}(2)$
	$(\mathcal{M}, \mathcal{M})$	−0.1045 (0)	−0.2992 (−0.3399)	0.0813 (0)
	$(\mathcal{M}, \mathcal{E})_+$	−0.0220 (0.0069)	−0.2968 (−0.4440)	−0.0020 (−0.0052)
	$(\mathcal{M}, \mathcal{E})_-$	−0.0358 (−0.0069)	−0.2720 (−0.1280)	0.0084 (0.0052)
	$(\mathcal{E}, \mathcal{E})$	0.1382 (0)	−0.3205 (−0.3277)	0.0143 (0)
	$(\mathcal{M}, \mathcal{M})$	−0.1963 (0)	0.0981 (0)	0.0981 (0)
	$(\mathcal{M}, \mathcal{E})_+$	−0.3241 (−0.0340)	0.1621 (0.0170)	0.1621 (0.0170)
	$(\mathcal{M}, \mathcal{E})_-$	−0.0011 (0)	−0.0017 (0)	−0.0005 (0)
	$(\mathcal{E}, \mathcal{E})$	−0.2073 (0)	0.1036 (0)	0.1036 (0)
	$(\mathcal{M}, \mathcal{M})$	−0.1549	0.1549	0.0774
	$(\mathcal{M}, \mathcal{E})_+$	−0.0555	0.0748	0.0513
	$(\mathcal{M}, \mathcal{E})_-$	0.0469	−0.0278	−0.0941
	$(\mathcal{E}, \mathcal{E})$	−0.1616	0.1616	0.0807

[a]The numbers in parentheses (where given) show the contributions from the transverse polarizations of the virtual gluon. Note that $(\mathcal{M}, \mathcal{E})_\pm = (1/\sqrt{2})[(\mathcal{M}, \mathcal{E}) \pm (\mathcal{E}, \mathcal{M})]$.

5.3.1. The Diquark System

Since the antiquark-antiquark interaction is, with trivial changes of the quantum numbers, identical to the quark-quark interaction, we will discuss here only the latter. The diquark can occupy the following irreducible representations of the various symmetry groups:

$$\begin{aligned} &SU(3)_{\text{color}}: && (1,0)\otimes(1,0)=(0,1)_{\mathrm{A}}\oplus(2,0)_{\mathrm{S}} \\ &SU(2)_{\text{spin}}: && \tfrac{1}{2}\otimes\tfrac{1}{2}=0_{\mathrm{A}}\oplus 1_{\mathrm{S}} \\ &SU(2)_{\text{isospin}}: && \tfrac{1}{2}\otimes\tfrac{1}{2}=0_{\mathrm{A}}\oplus 1_{\mathrm{S}} \end{aligned} \tag{5.19}$$

We indicate with a subscript whether the representation is symmetric (S) or antisymmetric (A) with respect to the interchange of the two quarks. The two-quark states that are consistent with the Fermi-Dirac statistics have the quantum numbers

$$\begin{aligned} (p,q)J^{\pi}I=(0,1)0^{+}0 \quad &(2,0)0^{+}1 \\ (0,1)1^{+}1 \quad &(2,0)1^{+}0 \end{aligned} \tag{5.20}$$

The two-body operator describing the interaction via one-gluon exchange between two quarks or antiquarks (Figs. 2a and 2b, respectively) is given by

$$\Delta_{12}=\mathbf{F}_1\cdot\mathbf{F}_2[\mu_{12}(0)+4\mu_{12}(1)\mathbf{S}_1\cdot\mathbf{S}_2] \tag{5.21}$$

This operator is diagonal in the two-particle space and has the matrix elements

$$\begin{aligned} \Delta &= \langle(p,q)J^{\pi}I|\Delta_{12}|(p,q)J^{\pi}I\rangle \\ &= \tfrac{1}{2}[C(p,q)-\tfrac{8}{3}]\{\mu(0)+2\mu(1)[J(J+1)-\tfrac{3}{2}]\} \end{aligned} \tag{5.22}$$

where $C(p,q)$ is the Casimir operator of $SU(3)_{\text{color}}$, given in equation (5.16). The matrix elements of Δ_{12} are shown in Figure 3 and also in Table V. Note that the color $\{\bar{3}\}=(0,1)$ interaction is twice as strong as the color $\{6\}=(2,0)$ interaction. Since the ordinary baryons are usually taken to be colorless three-quark states, only the color $\{\bar{3}\}$ interaction is accessible to experimental tests so far.

5.3.2. The Quark–Antiquark System

The quark–antiquark system can have the following color quantum numbers

$$SU(3)_{\text{color}}:\quad (1,0)\otimes(0,1)=(0,0)\oplus(1,1) \tag{5.23}$$

whereas the spin and isospin products correspond to the quark–quark case (5.19). Since the antiquark is distinguishable from the quark, the exclusion principle does not apply and the eight possible states are

$$\begin{aligned}(p,q)J^{\pi}I=(0,0)0^{-}0 &\quad (1,1)0^{-}0\\ (0,0)0^{-}1 &\quad (1,1)0^{-}1\\ (0,0)1^{-}0 &\quad (1,1)1^{-}0\\ (0,0)1^{-}1 &\quad (1,1)1^{-}1\end{aligned} \tag{5.24}$$

The diagrams contributing to the quark–antiquark interaction are the one-gluon exchange (Figure 2c) and the virtual annihilation into a gluon

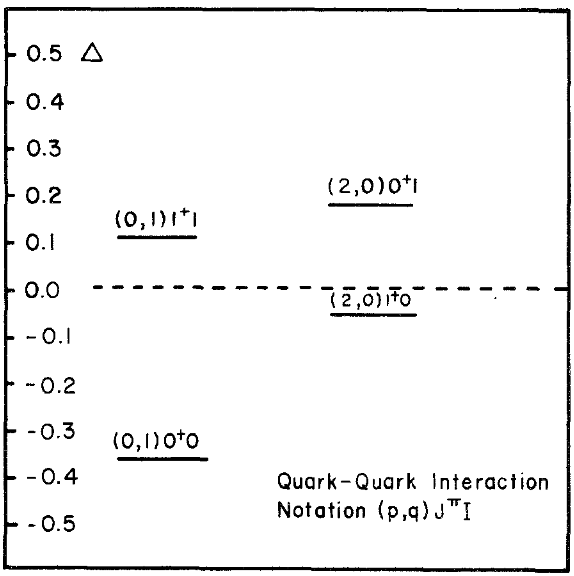


Fig. 3. The dimensionless energy shifts Δ for an interacting quark–quark system. The quarks occupy the $1s_{1/2}$ mode. See also Table V.

Table V. The Dimensionless Energy Shifts Δ for an Interacting Quark-Quark System[a]

$(p, q)J^{\pi}I$	
$(0, 1)0^{+}0$	−0.3605
$(0, 1)1^{+}1$	0.1115
$(2, 0)0^{+}1$	0.1803
$(2, 0)1^{+}1$	−0.0557

[a]The quarks occupy the $1s_{1/2}$ mode. See also Figure 3.

(Figure 2d). The corresponding two-body operator is

$$\begin{aligned}\Delta_{12} = {} & \mathbf{F}_1 \cdot \mathbf{F}_2[\mu_{12}(0) + 4\mu_{12}(1)\mathbf{S}_1 \cdot \mathbf{S}_2] \\ & + \nu_{12}(1)[\tfrac{1}{4} - \mathbf{T}_1 \cdot \mathbf{T}_2][\mathbf{F}_1 \cdot \mathbf{F}_2 + \tfrac{4}{3}][\mathbf{S}_1 \cdot \mathbf{S}_2 + \tfrac{3}{4}] \qquad (5.25)\end{aligned}$$

with the matrix elements

$$\begin{aligned}\Delta &= \langle (p, q)J^{\pi}I | \Delta_{12} | (p, q)J^{\pi}I \rangle \\ &= \tfrac{1}{2}[C(p, q) - \tfrac{8}{3}]\{\mu(0) + 2\mu(1)[J(J+1) - \tfrac{3}{2}]\} \\ &\quad + \tfrac{1}{4}\nu(1)C(p, q)J(J+1)[1 - \tfrac{1}{2}I(I+1)] \qquad (5.26)\end{aligned}$$

In order to avoid confusion, we use the symbol $\nu(1)$ instead of $\mu(1)$ for the nonvanishing coefficient of the annihilation diagram, so that $\nu(1) = 0.1875$. The matrix elements Δ are shown in Figure 4. Table VI contains the contributions to Δ from the gluon exchange and the annihilation diagrams. Note that the second term in equation (5.25) or equation (5.26), which represents the annihilation diagram, vanishes unless the quark-antiquark pair carries the quantum number of the gluon $(p, q)J^{\pi}I = (1, 1)1^{-}0)$. For this case we find that the annihilation diagram is ten times bigger in magnitude than the one-gluon exchange diagram and of opposite sign. Without this annihilation diagram the two states $(p, q)J^{\pi}I = (1, 1)1^{-}0$ (color octet ω-meson) and $(p, q)J^{\pi}I = (1, 1)1^{-}1$ (color octet ρ-meson) would be degenerate. The possibility of virtually decaying into a gluon makes the ω^{8} heavier than the ρ^{8}. However, the pionlike state $(p, q)J^{\pi}I = (0, 0)0^{-}1$ remains degenerate with the etalike state $(p, q)J^{\pi}I = (0, 0)0^{-}0$, in contradiction to experimental findings. Of course, we know that the π- and η-mesons are, in the real world, more complex objects than just quark-antiquark systems consisting of up and down quarks.

5.3.3. The Quark-Gluon System

As already in the quark-quark case, the antiquark-gluon interaction leads to the same matrix elements for corresponding quantum numbers as

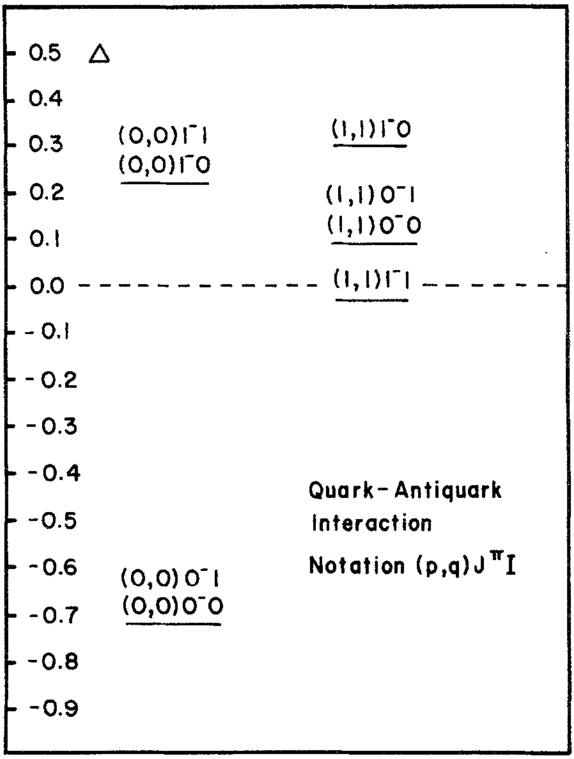
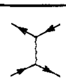


Fig. 4. The dimensionless energy shifts Δ for an interacting quark-antiquark system. The quark and the antiquark occupy the $1s_{1/2}$ modes. See also Table VI.

Table VI. The Dimensionless Energy Shifts Δ for an Interacting Quark-Antiquark System[a]

$(p, q)JJ^{\pi}I$			Total
$(0,0)0^{-}0$ $(0,0)0^{-}1$	−0.7211	0	−0.7211
$(0,0)1^{-}0$ $(0,0)1^{-}1$	0.2229	0	0.2229
$(1,1)0^{-}0$ $(1,1)0^{-}1$	0.0901	0	0.0901
$(1,1)1^{-}0$	−0.0279	0.2813	0.2534
$(1,1)1^{-}1$		0	−0.0279

[a] We have separated the contributions from the one-gluon-exchange and the annihilation diagrams for a quark and antiquark in the $1s_{1/2}$ mode. See also Figure 4.

the quark-gluon interaction, so that we consider here only the latter. The quark-gluon system can couple to the following quantum numbers:

$$\begin{aligned} SU(3)_{\text{color}}&: \quad (1,0)\otimes(1,1)=(1,0)\oplus(0,2)\oplus(2,1)\\ SU(2)_{\text{spin}}&: \quad \tfrac{1}{2}\otimes 1=\tfrac{1}{2}\oplus\tfrac{3}{2} \qquad (5.27)\\ SU(2)_{\text{isospin}}&: \quad \tfrac{1}{2}\otimes 0=\tfrac{1}{2} \end{aligned}$$

Of course, we do not need to symmetrize or antisymmetrize the direct product states; the possible quantum numbers of the quark-gluon system are

$$\begin{aligned} (p,q)J^{\pi}I: \quad &(1,0)\tfrac{1}{2}^{+}\tfrac{1}{2} \quad (0,2)\tfrac{1}{2}^{+}\tfrac{1}{2} \quad (2,1)\tfrac{1}{2}^{+}\tfrac{1}{2}\\ &(1,0)\tfrac{3}{2}^{+}\tfrac{1}{2} \quad (0,2)\tfrac{3}{2}^{+}\tfrac{1}{2} \quad (2,1)\tfrac{3}{2}^{+}\tfrac{1}{2} \qquad (5.28) \end{aligned}$$

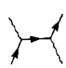
The interaction between these two particles receives contributions from the direct (D) and exchange (E) Compton diagrams and the one-gluon exchange, shown in Figures 2e–2g, respectively. The corresponding two-body operators can be read off Table I; the matrix elements of their sum are

$$\begin{aligned} \Delta &= \langle (p,q)J^{\pi}I|\Delta_{12}|(p,q)J^{\pi}I\rangle\\ &= \tfrac{1}{18}(4F^2-1)\{\tfrac{1}{2}\mu^{D}(1)[\tfrac{15}{4}-J(J+1)]+\tfrac{1}{2}\mu^{D}(2)[J(J+1)-\tfrac{3}{4}]\}\\ &\quad +\tfrac{1}{18}(4F^2+6F-1)\{\tfrac{1}{2}\mu^{E}(1)[J(J+1)-\tfrac{7}{4}]+\tfrac{1}{2}\mu^{E}(2)[\tfrac{19}{4}-J(J+1)]\}\\ &\quad +F\{\mu(0)+\tfrac{1}{2}\mu(1)[J(J+1)-\tfrac{11}{4}]\} \qquad (5.29) \end{aligned}$$

Here, F is related to the Casimir operator of $SU(3)_{\text{color}}$ by

$$F=\langle (p,q)|\mathbf{F}_1\cdot\mathbf{F}_2|(p,q)\rangle=\tfrac{1}{2}[C(p,q)-\tfrac{13}{3}] \qquad (5.30)$$

Table VII. The Dimensionless Energy Shifts Δ for an Interacting Quark-Gluon System[a]

$(p,q)J^{\pi}I$				Total
$(1,0)\frac{1}{2}^{+}\frac{1}{2}$	0.3744	−0.0325	−0.7271	−0.3852
$(1,0)\frac{3}{2}^{+}\frac{1}{2}$	0.0421	0.0125	0.3755	0.4301
$(0,2)\frac{1}{2}^{+}\frac{1}{2}$	0	−0.0975	−0.2424	−0.3399
$(0,2)\frac{3}{2}^{+}\frac{1}{2}$	0	0.0375	0.1252	0.1627
$(2,1)\frac{1}{2}^{+}\frac{1}{2}$	0	0.0975	0.2424	0.3399
$(2,1)\frac{3}{2}^{+}\frac{1}{2}$	0	−0.0375	−0.1252	−0.1627

[a] We show the contributions from the various Feynman diagrams for a quark in the $1s_{1/2}$ and a gluon in the $1\mathscr{M}_1$ mode. See also Figure 5.

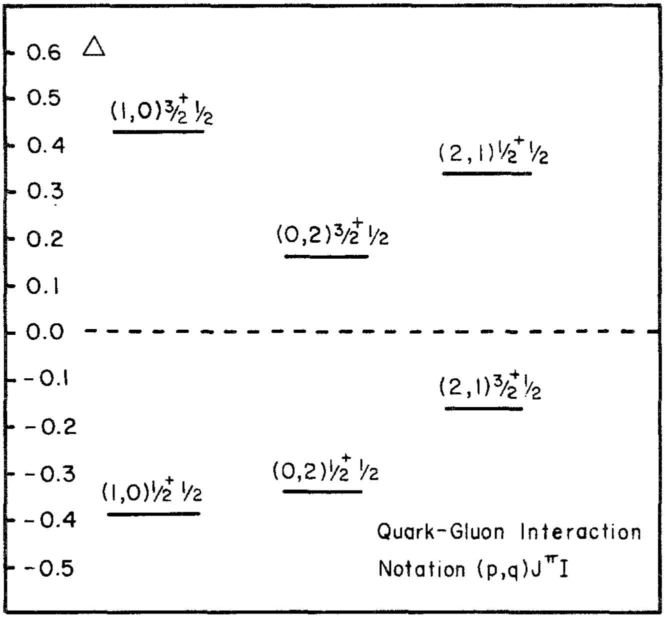


Fig. 5. The dimensionless energy shifts Δ for an interacting quark-gluon system. The quark occupies the $1s_{1/2}$ and the gluon the $1\mathcal{M}_1$ mode. See also Table VII.

In equation (5.29), the second, third, and fourth lines correspond to the direct and the exchange Compton diagrams and the gluon exchange, respectively. The shifts of the two-particle energy levels due to these three interactions can be found in Figure 5. As can be seen in Table VII, the energy shifts are dominated by the contribution from the gluon exchange between the quark and the gluon.

5.3.4. The Gluon–Gluon System

Two gluons can be accommodated in the following irreducible representations of the groups $SU(3)_{\text{color}}$ and $SU(2)_{\text{spin}}$:

$$\begin{aligned} SU(3)_{\text{color}}:\quad & (1,1)\otimes(1,1)=(0,0)_{\text{S}}\oplus(1,1)_{\text{S}}\oplus(1,1)_{\text{A}}\oplus(3,0)_{\text{A}} \\ & \qquad (0,3)_{\text{A}}\oplus(2,2)_{\text{S}} \\ SU(2)_{\text{spin}}:\quad & 1\otimes 1=0_{\text{S}}\oplus 1_{\text{A}}\oplus 2_{\text{S}} \end{aligned} \tag{5.31}$$

We have again indicated the symmetry property of the representations with respect to the interchange of the two particles. Since the gluons are bosons, the Bose–Einstein statistics allows only the following nine totally symmetric two-particle states:

$$\begin{array}{lllll} (p,q)J^\pi: & (0,0)0^+ & (1,1)0^+ & (3,0)1^+ & (2,2)0^+ \\ & (0,0)2^+ & (1,1)1^+ & (0,3)1^+ & (2,2)2^+ \\ & & (1,1)2^+ & & \end{array} \tag{5.32}$$

The two-body operators that describe the gluon–gluon interactions through gluon exchange, annihilation, and the four-gluon vertex are given in all three cases by (Buser, 1983, Hess and Viollier, 1986, 1988)

$$\Delta_{12} = \mathbf{F}_1 \cdot \mathbf{F}_2[\rho_{12}(0) + \rho_{12}(1)\mathbf{S}_1 \cdot \mathbf{S}_2 + \rho_{12}(2)(\mathbf{S}_1 \cdot \mathbf{S}_2)^2] \tag{5.33}$$

and have the matrix elements

$$\begin{aligned} \Delta &= \langle (p,q)J^\pi | \Delta_{12} | (p,q)J^\pi \rangle \\ &= \tfrac{1}{2}[C(p,q) - 6]\{\rho(0) + \tfrac{1}{2}\rho(1)[J(J+1) - 4] + \tfrac{1}{4}\rho(2)[J(J+1) - 4]^2\} \end{aligned} \tag{5.34}$$

The coefficients $\rho(k)$ can be read off in Table IV for the various diagrams. The resulting matrix elements Δ are shown in Figure 6; Table VIII indicates again the contribution to Δ from the individual diagrams. As compared to the previous cases in Sections 5.3.1–5.3.3, the two-particle energy shifts due to the interactions in second-order perturbation theory are much larger for the gluon–gluon system. In particular, the state carrying the quantum numbers of the vacuum, $(p,q)J^\pi = (0,0)0^+$, is so much lowered in energy that it could become degenerate with the vacuum. Of course, whether this degeneracy is present depends on the value of the strong "hyperfine" constant α_s and the (up to now not reliably calculated) gluon self-energies, among others. Due to the vanishing of $\mathbf{F}_1 \cdot \mathbf{F}_2$ in the $SU(3)_{\text{color}}$ representations $(p,q) = (3,0)$, $(0,3)$, the two gluons do not interact in the decuplet and antidecuplet cases.

Our results are consistent with those of Carlson *et al.* (1983a) for their gluon exchange diagram denoted by $3g$, corresponding to the diagram of Figure 2 in this work, if only the transverse polarization modes of the virtual gluon are taken into account. Using the transformation of the coefficients as given by Carlson *et al.* (1983a), we can further show agreement of our four-gluon vertex with theirs for the cases $J = 0$ and $J = 2$, but not for $J = 1$ (J being the spin of the gluon pair) (note the different naming of the gluon polarization modes). As for the Coulomb interaction, which is the gluon exchange between two gluons mediated by the scalar and longitudinal polarizations, we disagree with them on the value of their coefficient $\tilde{c}$, even as corrected in Carlson *et al.* (1983b). The $\tilde{c}$ corresponds to $-\rho(0) - \rho(2)$ in this paper.

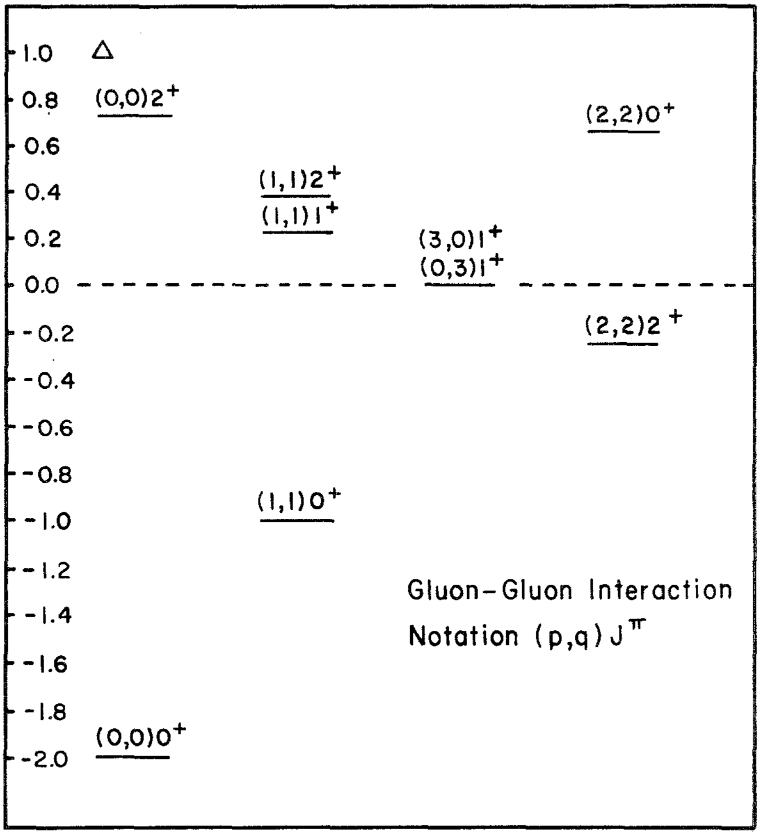


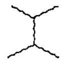
Fig. 6. The dimensionless energy shifts Δ for an interacting gluon-gluon system. The gluons occupy the $1\mathscr{M}_1$ mode. See also Table VIII.

6. CONCLUSIONS AND OUTLOOK

In this paper we have studied a version of quantum chromodynamics in which the quarks, gluons, and ghosts are forced to move in a static and spherically symmetric cavity. The confinement of these particles was achieved by imposing on the field operators the linear boundary conditions of the MIT bag model. All nondiverging Feynman diagrams obtained in perturbation expansion of this quantum field theory have been calculated in the Feynman gauge up to order α_s. In most of the cases where we can compare our results to previous calculations performed in other gauges we found agreement.

Of course, many open questions remain to be addressed before one can actually claim that quantum chromodynamics in a static and spherically

Table VIII. The Dimensionless Energy Shifts Δ for an Interacting Gluon–Gluon System[a]

$(p, q)J^{\pi}$				Total
$(0, 0)0^+$	−2.4575	0	0.4646	−1.9929
$(0, 0)2^+$	0.9672	0	−0.2323	0.7349
$(1, 1)0^+$	−1.2289	0	0.2323	−0.9966
$(1, 1)1^+$	−0.4140	0.2944	0.3485	0.2288
$(1, 1)2^+$	0.4836	0	−0.1161	0.3675
$(3, 0)1^+$, $(0, 3)1^+$	0	0	0	0
$(2, 2)0^+$	0.8191	0	−0.1549	0.6642
$(2, 2)2^+$	−0.3224	0	0.0774	−0.2450

[a]We show the contributions from the various Feynman diagrams for gluons in the $1\mathcal{M}_1$ mode. See also Figure 6.

symmetric cavity is a consistent quantum field theory. First, the self-energies of the quarks and gluons, which appear already to order α_s, must be calculated in a reliable way. A promising step in solving these renormalization problems in the cavity has been made with the development of the multiple reflection formalism, which has been applied recently to the calculation of the quark self-energies (Hansson and Jaffe, 1983; Goldhaber *et al.*, 1983, 1986). The self-energies of the gluon, however, still remain to be evaluated, in order to make sure that the breaking of translational and Lorentz invariance through the boundary conditions will not spoil the renormalizability of the quantum field theory at least to order α_s.

The formalism presented here in the Feynman gauge with the Faddeev–Popov ghost fields is best suited for the renormalization of Feynman graphs. Due to the locality of the quantum field theory in this covariant gauge, the short-distance singularities are softer than in other gauges, e.g., the Coulomb gauge. In this context, it is also interesting to note that the short-distance singularities are not affected by the boundary conditions and, more important we do not have to worry about infrared divergences in "bagged" quantum chromodynamics. For the proof of renormalizability, it is also encouraging to know that the boundary conditions preserve the Becchi–Rouet–Stora symmetry of the theory.

Besides studying the renormalizability of the theory, it is important to assess the validity of the perturbation expansion of this "bagged" quantum field theory by actually calculating diagrams of higher order in α_s (Stoddart and Viollier, 1988). One might think that with coupling strengths α_s of order unity it is hopeless to try a perturbation expansion. This need not be the case in "bagged" QCD, which differs in many respects from QCD in "infinite space." First, the quarks, gluons, and ghosts must occupy cavity modes of a certain energy. Thus, it is not possible that a quark can radiate off a gluon with arbitrary small energy in "bagged" QCD. Second, all expressions we have deduced so far look like old-fashioned perturbation theory. Thus, it is not only the strong fine structure constant α_s that determines the convergence of the perturbation expansion, but rather a combination of α_s, energy denominators, and vertex integrals that usually decrease very rapidly for the higher cavity modes.

Quantum chromodynamics will have to predict some numbers in the low-energy region that can be verified with the precise low-energy experiments to at least three or four digits, such as the Lamb shift or the anomalous magnetic moment of the electron in quantum electrodynamics. It is perfectly possible that these questions can only be addressed once the confinement and the center-of-mass problem have been solved. It is also possible, however, that for certain precisely measured low-energy quantities, e.g., ratios or splittings, it does not matter in which way confinement was achieved and how the center-of-mass corrections are done. In this case, one could use the low-energy data as a precise test of quantum chromodynamics.

Quantum chromodynamics also predicts the existence of hadronic matter that is not composed of three quarks and a quark–antiquark pair primarily. In spite of the tremendous effort put into the clarification of this crucial issue, the experimental evidence for these exotic hadrons is still rather weak. We should be able, however, to understand the properties of these hadronic states if they exist. Conversely, if they do not exist, we will have to find out why, out of the tremendous number of possible color singlet states consisting of quarks, antiquarks, and gluons, only quark–antiquark and three-quark states are realized in nature.

The ultimate and, from the theoretical point of view, most challenging and exciting problem in QCD is to discover how the original "infinite-space" theory manages to confine the color-carrying particles in a finite region of space. We will have to find out whether it is due to a breakdown of the perturbative vacuum or to as yet undiscovered topological features of this non-Abelian gauge theory. In this context, it is perhaps important to realize that nuclei can and should be used as a laboratory to test our ideas about confinement.

APPENDIX A. THE CAVITY MODES

A1. Quarks

Here we derive the cavity modes of the quarks in a spherically symmetric and static cavity. The explicit solutions of the time independent Dirac equation

$$(-i\boldsymbol{\gamma}\cdot\boldsymbol{\nabla}+m_f)u_n(\mathbf{r})=\varepsilon_n\gamma^0u_n(\mathbf{r}) \tag{A1}$$

where ε_n is the energy and m_f the mass of the quark, are given by the Dirac spinors

$$u_n(\mathbf{r})=\begin{pmatrix} g_n(r)\chi_\kappa^\mu(\hat{\mathbf{r}}) \\ if_n(r)\chi_{-\kappa}^\mu(\hat{\mathbf{r}})\end{pmatrix} \tag{A2}$$

Of course, the adjoint spinor is defined as

$$\bar{u}_n(\mathbf{r})=u_n^+(\mathbf{r})\gamma^0 \tag{A3}$$

Here $\chi_\kappa^\mu(\hat{\mathbf{r}})$ denote the usual two-component spherical spinors and n stands for the flavor, radial, and Dirac quantum numbers and sometimes also includes the magnetic quantum number as well, i.e.,

$$n=\{f,\nu,\kappa,(\mu)\} \tag{A4}$$

The radial functions $g_n(r)$ and $f_n(r)$ are given in terms of the spherical Bessel functions by

$$g_n(r)=\frac{\mathcal{N}_n}{R^{3/2}}j_l(p_nr) \tag{A5}$$

$$f_n(r)=\frac{\mathcal{N}_np_n\operatorname{sgn}\kappa}{R^{3/2}(\varepsilon_n+m_f)}j_{\bar{l}}(p_nr) \tag{A6}$$

where R is the radius of the cavity. Here, the total and orbital angular momenta j, l, and $\bar{l}$ are defined as functions of the quantum number κ:

$$j(\kappa)=|\kappa|-\tfrac{1}{2} \tag{A7}$$

$$l(\kappa)=j(\kappa)+\tfrac{1}{2}\operatorname{sgn}\kappa \tag{A8}$$

$$\bar{l}(\kappa)=j(\kappa)-\tfrac{1}{2}\operatorname{sgn}\kappa \tag{A9}$$

Finally, the symbol ν denotes the radial quantum number, with $\nu>0$ for positive and $\nu<0$ for negative energies, respectively. Of course, a complete set of Dirac eigenfunctions must include the negative energy states as well. The quark momenta p_n are determined by the linear boundary condition of the MIT bag model

$$(i\boldsymbol{\gamma}\cdot\hat{\mathbf{r}}+1)u_n(\mathbf{r})|_{r=R}=0 \tag{A10}$$

or, equivalently, by the solutions of the equation

$$j_l(x_n)+\frac{x_n\operatorname{sgn}\kappa}{\omega_n+\zeta_f}j_{\bar{l}}(x_n)=0 \tag{A11}$$

Here, we have introduced the dimensionless energy, momentum, and mass parameters, respectively,

$$\omega_n = \varepsilon_n R = \text{sgn}\ \nu_+(x_n^2 + \zeta_f^2)^{1/2} \tag{A12}$$

$$x_n = p_n R \tag{A13}$$

$$\zeta_f = m_f R \tag{A14}$$

The normalization constant $\mathcal{N}_n$ is given by

$$\mathcal{N}_n^{-2} = [2\omega_n(\omega_n + \kappa) + \zeta_f]\left[\frac{j_l(x_n)}{x_n}\right]^2 \tag{A15}$$

The solutions of the Dirac equation (A1) satisfying the boundary condition (A10) represent a complete and orthonormal set of Dirac spinors in the cavity, i.e.,

$$\int u_n^+(\mathbf{r}) u_{n'}(\mathbf{r})\ d^3 r = \delta_{nn'} \tag{A16}$$

$$\sum_{\nu\kappa\mu} u_{n\alpha}^*(\mathbf{r}) u_{n\beta}(\mathbf{r}') = \delta_{\alpha\beta}\delta^{(3)}(\mathbf{r} - \mathbf{r}') \tag{A17}$$

where $u_{n\alpha}(\mathbf{r})$ denotes the component α of the Dirac spinor $u_n(\mathbf{r})$.

A2. Gluons and Ghosts

Let us now determine, in the Feynman gauge ($\lambda = 1$), the cavity modes of the gluon in a spherically symmetric and static cavity. The explicit solutions of the time-independent d'Alembert equations

$$[\Delta + (\Omega_m^{0^2})] a_m^0(\mathbf{r}) = 0 \tag{A18}$$

$$[\Delta + (\Omega_m^{\Sigma})^2] \mathbf{a}_m^{\Sigma}(\mathbf{r}) = 0; \qquad \Sigma = \mathcal{L}, \mathcal{M}, \mathcal{E} \tag{A19}$$

where Ω_m^{Σ} denotes the energy eigenvalue of the gluon, are given by the Hansen functions, which include the scalar

$$a_m^0(\mathbf{r}) = \frac{\mathcal{N}_m^0}{R^{3/2}}\, i j_J(\Omega_m^0 r)\, Y_{JM}(\hat{\mathbf{r}}), \qquad J \geq 0 \tag{A20}$$

and longitudinal multipole fields

$$\mathbf{a}_m^{\mathcal{L}}(\mathbf{r}) = \frac{\mathcal{N}_m^{\mathcal{L}}}{R^{3/2}} \frac{1}{\Omega_m^{\mathcal{L}}} \nabla j_J(\Omega_m^{\mathcal{L}} r)\, Y_{JM}(\hat{\mathbf{r}}), \qquad J \geq 0 \tag{A21}$$

Here, $m = \{N, J, (M)\}$ denotes a complete set of radial and angular momentum quantum numbers and Σ stands for the polarization of the gluon. The transverse magnetic and electric modes are given by

$$\mathbf{a}_m^{\mathcal{M}}(\mathbf{r}) = \frac{\mathcal{N}_m^{\mathcal{M}}}{R^{3/2}} \frac{\mathbf{L}}{[J(J+1)]^{1/2}} (j_J(\Omega_m^{\mathcal{M}} r) Y_{JM}(\mathbf{r})), \qquad J \geq 1 \tag{A22}$$

$$\mathbf{a}_m^{\mathcal{E}}(\mathbf{r}) = \frac{\mathcal{N}_m^{\mathcal{E}}}{R^{3/2}} \frac{1}{i\Omega_m^{\mathcal{E}}} \nabla \times \frac{\mathbf{L}}{[J(J+1)]^{1/2}} (j_J(\Omega_m^{\mathcal{E}} r) Y_{JM}(\hat{\mathbf{r}})), \qquad J \geq 1 \tag{A23}$$

respectively. The eigenvalues of the gluon Ω_m^{Σ} are determined by the linear boundary conditions of the MIT bag model

$$\hat{\mathbf{r}} \cdot \nabla a_m^0(\mathbf{r})|_{r=R} = 0 \tag{A24}$$

$$\hat{\mathbf{r}} \cdot \mathbf{a}_m^{\Sigma}(\mathbf{r})|_{r=R} = 0 \tag{A25}$$

$$\hat{\mathbf{r}} \times (\nabla \times \mathbf{a}_m^{\Sigma}(\mathbf{r}))|_{r=R} = 0; \qquad \Sigma = \mathcal{L}, \mathcal{M}, \mathcal{E} \tag{A26}$$

For the various multipole fields, we then arrive at three eigenvalue equations

$$\frac{d}{dr} j_J(\Omega_m^0 r)|_{r=R} = 0 \tag{A27}$$

$$\frac{d}{dr} [r j_J(\Omega_m^{\mathcal{M}} r)]|_{r=R} = 0 \tag{A28}$$

$$j_J(\Omega_m^{\mathcal{E}} r)|_{r=R} = 0 \tag{A29}$$

since the scalar and longitudinal multipole fields satisfy the same eigenvalue equation, i.e., $\Omega_m^0 = \Omega_m^{\mathcal{L}}$. The corresponding normalization constants are

$$[\mathcal{N}_m^0]^{-2} = \tfrac{1}{2} j_J^2(\mathcal{Y}_m^0)[1 - J(J+1)/(\mathcal{Y}_m^0)^2] = [\mathcal{N}_m^{\mathcal{L}}]^{-2} \tag{A30}$$

$$[\mathcal{N}_m^{\mathcal{M}}]^{-2} = \tfrac{1}{2} j_J^2(\mathcal{Y}_m^{\mathcal{M}})[1 - J(J+1)/(\mathcal{Y}_m^{\mathcal{M}})^2] \tag{A31}$$

$$[\mathcal{N}_m^{\mathcal{E}}]^{-2} = \tfrac{1}{2} j_{J+1}^2(\mathcal{Y}_m^{\mathcal{E}}) \tag{A32}$$

where we have introduced the dimensionless energy parameters

$$\mathcal{Y}_m^{\Sigma} = \Omega_m^{\Sigma} R; \qquad \Sigma = 0, \mathcal{L}, \mathcal{M}, \mathcal{E} \tag{A33}$$

For compactness of the notation we introduce the functions

$$a_m^{\mu\Sigma}(\mathbf{r}) = \begin{cases} a_m^0(\mathbf{r}) & \text{for } \mu = \Sigma = 0 \\ [\mathbf{a}_m^{\Sigma}(\mathbf{r})]^{\mu} & \text{for } \mu = 1, 2, 3 \text{ and } \Sigma = \mathcal{L}, \mathcal{M}, \mathcal{E} \\ 0 & \text{for all other } \mu \text{ and } \Sigma \text{ values} \end{cases} \tag{A34}$$

The solutions of the d'Alembert equations (A18) and (A19) that satisfy the boundary conditions (A24)–(A26) are orthonormal, i.e.,

$$\int a_{\mu m}^{\Sigma}(\mathbf{r})^* a_{m'}^{\mu\Sigma'}(\mathbf{r})\, d^3r = g^{\Sigma\Sigma'} \delta_{mm'} \tag{A35}$$

and they also satisfy the completeness relation

$$\sum_{\Sigma m} g^{\Sigma\Sigma} a_m^{\mu\Sigma}(\mathbf{r})^* a_m^{\nu\Sigma}(\mathbf{r}') = g^{\mu\nu} \delta^{(3)}(\mathbf{r} - \mathbf{r}') \tag{A36}$$

Here we have introduced the metric tensor

$$\begin{aligned} g^{00} &= -g^{\mathscr{L}\mathscr{L}} = -g^{\mathscr{M}\mathscr{M}} = -g^{\mathscr{E}\mathscr{E}} = 1 \\ g^{\Sigma\Sigma'} &= 0 \qquad \text{if} \quad \Sigma \neq \Sigma' \end{aligned} \tag{A37}$$

The cavity modes $\mathbf{a}_m^\Sigma(\mathbf{r})$ can also be written in terms of the spherical Bessel functions and the vector spherical harmonics as

$$\mathbf{a}_m^\Sigma(\mathbf{r}) = \frac{\mathcal{N}_m^\Sigma}{R^{3/2}} \frac{1}{(2J+1)^{1/2}} \sum_{L=|J-1|}^{J+1} \alpha_{JL}^\Sigma j_L(\Omega_m^\Sigma r) \mathbf{Y}_{JLM}(\hat{\mathbf{r}}) \tag{A38}$$

A similar expansion holds for the curl of the cavity modes

$$\nabla \times \mathbf{a}_m^\Sigma(\mathbf{r}) = -\mathrm{i} \frac{\Omega_m^\Sigma \mathcal{N}_m^\Sigma}{R^{5/2}} \frac{1}{(2J+1)^{1/2}} \sum_{L=|J-1|}^{J+1} \beta_{JL}^\Sigma j_L(\Omega_m^\Sigma r) \mathbf{Y}_{JLM}(\hat{\mathbf{r}}) \tag{A39}$$

The only nonvanishing coefficients α_{JL}^Σ and β_{JL}^Σ are given by

$$\begin{aligned} &\alpha_{J,J+1}^{\mathscr{L}} = (J+1)^{1/2}, \qquad \alpha_{J,J-1}^{\mathscr{L}} = J^{1/2} \\ &\alpha_{J,J}^{\mathscr{M}} = (2J+1)^{1/2}, \qquad \beta_{J,J+1}^{\mathscr{M}} = J^{1/2}, \qquad \beta_{J,J-1}^{\mathscr{M}} = -(J+1)^{1/2} \\ &\alpha_{J,J+1}^{\mathscr{E}} = -J^{1/2}, \qquad \alpha_{J,J-1}^{\mathscr{E}} = (J+1)^{1/2}, \qquad \beta_{J,J}^{\mathscr{E}} = (2J+1)^{1/2} \end{aligned} \tag{A40}$$

Under complex conjugation the gluon modes $a_m^{\mu\Sigma}(\mathbf{x})$ transform according to

$$a_m^{\mu\Sigma}(\mathbf{x})^* = \eta_\Sigma (-1)^M a_{m^*}^{\mu\Sigma}(\mathbf{x}) \tag{A41}$$

where the set of quantum numbers m^* is defined by

$$m^* = \{N, J, (-M)\} \tag{A42}$$

and the phase η_Σ stands for

$$\eta_\Sigma = \begin{cases} +1 & \text{for} \quad \Sigma = \mathscr{L}, \mathscr{E} \\ -1 & \text{for} \quad \Sigma = 0, \mathscr{M} \end{cases} \tag{A43}$$

Finally the cavity modes and eigenvalues for the ghost fields in a spherically symmetric and static cavity are also given by the scalar modes $a_m^0(\mathbf{r})$ and the scalar or longitudinal eigenvalues Ω_m^0, since they satisfy the d'Alembert equation (A18) and the boundary condition (A24).

APPENDIX B. THE FEYNMAN PROPAGATORS

B1. The Quark Propagator

We now turn to the evaluation of the Feynman propagators, which are given in terms of vacuum expectation values of time-ordered products of two field operators. There are two nonvanishing Feynman propagators that involve the quark field operators. Using the expansions (4.15) and (4.18) and the anticommutation relations (4.19), we easily obtain

$$\begin{aligned}&\langle\hat{0}|T(\hat{\bar{\psi}}^{+}_{cf\alpha}(x)\hat{\psi}_{c'f'\alpha'}(y))|\hat{0}\rangle\\&\quad=\delta_{cc'}\delta_{ff'}\sum_{\substack{\kappa\mu\\ \nu>0}}[\bar{u}_{-n\alpha}(\mathbf{x})u_{-n\alpha'}(\mathbf{y})\Theta(x^0-y^0)\\&\qquad-\bar{u}_{n\alpha}(\mathbf{x})u_{n\alpha'}(\mathbf{y})\Theta(y^0-x^0)]\exp(-i\varepsilon_n|x^0-y^0|)\end{aligned}\tag{B1}$$

The other nonvanishing vacuum expectation value is

$$\begin{aligned}&\langle\hat{0}|T(\hat{\psi}_{cf\alpha}(x)\hat{\bar{\psi}}^{+}_{c'f'\alpha'}(y))|\hat{0}\rangle\\&\quad=\delta_{cc'}\delta_{ff'}\sum_{\substack{\kappa\mu\\ \nu>0}}[u_{n\alpha}(\mathbf{x})\bar{u}_{n\alpha'}(\mathbf{y})\Theta(x^0-y^0)\\&\qquad-u_{-n\alpha}(\mathbf{x})\bar{u}_{-n\alpha'}(\mathbf{y})\Theta(y^0-x^0)]\exp(-i\varepsilon_n|x^0-y^0|)\end{aligned}\tag{B2}$$

In equations (B1) and (B2) we have made use of the step function, which is defined by

$$\Theta(x^0)=\begin{cases}0 & x^0<0\\ 1 & x^0>0\end{cases}\tag{B3}$$

B2. The Ghost Propagator

Based on the expansions (4.23) and (4.24) and the anticommutation relations (4.25), we arrive at Feynman propagators involving the ghost field operators, which are given by

$$\begin{aligned}&\langle\hat{0}|T(\hat{\omega}_a(x)\hat{\chi}_b(y))|\hat{0}\rangle=-\langle\hat{0}|T(\hat{\chi}_a(x)\hat{\omega}_b(y))|\hat{0}\rangle\\&\qquad=i\delta_{ab}\sum_{\substack{JM\\ N>0}}\frac{1}{2\Omega^0_m}a^0_m(\mathbf{x})a^0_m(\mathbf{y})^*\exp(-i\Omega^0_m|x^0-y^0|)\end{aligned}\tag{B4}$$

These two propagators are the only nonvanishing vacuum expectation values we can build of two ghost field operators. The Feynman propagators involving the derivative of $\hat{\chi}_a$ are easily evaluated by calculating the derivatives of equation (B4) with respect to x and y, respectively, i.e.,

$$\langle\hat{0}|T\left(\frac{\partial}{\partial x_\mu}\hat{\chi}_a(x)\hat{\omega}_b(y)\right)|\hat{0}\rangle=\frac{\partial}{\partial x_\mu}\langle\hat{0}|T(\hat{\chi}_a(x)\hat{\omega}_b(y))|\hat{0}\rangle\tag{B5}$$

$$\langle\hat{0}|T\left(\hat{\omega}_a(x)\frac{\partial\hat{\chi}_b(y)}{\partial y_\mu}\right)|\hat{0}\rangle=\frac{\partial}{\partial y_\mu}\langle\hat{0}|T(\hat{\omega}_a(x)\hat{\chi}_b(y))|\hat{0}\rangle\tag{B6}$$

B3. The Gluon Propagators

Based on the expansion (4.20) and the commutation relations (4.22), we arrive at the Feynman propagator for the gluon, which is given by

$$\langle\hat{0}|T(\hat{A}_a^\mu(x)\hat{A}_b^\nu(y))|\hat{0}\rangle = -\delta_{ab} \sum_{\substack{JM\Sigma \\ N>0}} \frac{g^{\Sigma\Sigma}}{2\Omega_m^\Sigma} a_m^{\mu\Sigma}(\mathbf{x}) a_m^{\nu\Sigma}(\mathbf{y})^* \exp(-i\Omega_m^\Sigma|x^0-y^0|) \tag{B7}$$

where the metric tensor $g^{\Sigma\Sigma}$ is defined by equation (A37).

We also need to know Feynman propagators involving derivatives of the field operators. In the case of a single derivative, these can be written as

$$\langle\hat{0}|T\left(\frac{\partial\hat{A}_a^\mu(x)}{\partial x_\rho}\hat{A}_b^\nu(y)\right)|\hat{0}\rangle = \frac{\partial}{\partial x_\rho}\langle\hat{0}|T(\hat{A}_a^\mu(x)\hat{A}_b^\nu(y))|\hat{0}\rangle \tag{B8}$$

In the case of two derivatives, however, we obtain

$$\langle\hat{0}|T\left(\frac{\partial\hat{A}_a^\mu(x)}{\partial x_\rho}\frac{\partial\hat{A}_b^\nu(y)}{\partial y_\sigma}\right)|\hat{0}\rangle = \frac{\partial^2}{\partial x_\rho\,\partial y_\sigma}\langle\hat{0}|T(\hat{A}_a^\mu(x)\hat{A}_b^\nu(y))|\hat{0}\rangle + i\delta_{ab}\delta_{\rho 0}\delta_{\sigma 0}g^{\mu\nu}\delta^{(4)}(x-y) \tag{B9}$$

Thus, here also we can in general take the derivatives of the vacuum expectation value of the time-ordered product of the field operators except for the case $\rho=\sigma=0$, where an additional delta term appears.

APPENDIX C. THE VERTEX INTEGRALS

C1. The Quark–Gluon Vertex

Here we evaluate the vertex integrals that describe the absorption (or emission) of a gluon by a quark (Figure 1a). These integrals are defined as

$$Q_{nn'm}^\Sigma = i\int \bar{u}_n(\mathbf{r})\gamma_\mu u_{n'}(\mathbf{r}) a_m^{\mu\Sigma}(\mathbf{r})\, d^3r \tag{C1}$$

In Appendix D we will also use [see (A41)-(A43)]

$$\tilde{Q}_{nn'm}^\Sigma = i\int \bar{u}_n(\mathbf{r})\gamma_\mu u_{n'}(\mathbf{r}) a_m^{\mu\Sigma*}(\mathbf{r})\, d^3r = (-1)^M\eta_\Sigma Q_{nn'm^*}^\Sigma = -Q_{n'nm}^\Sigma \tag{C1a}$$

where n and m stand for the quark and gluon quantum numbers, respectively, e.g.,

$$n=\{f,\nu,\kappa,(\mu)\} \tag{C2}$$

$$m=\{N,J,(M)\},\qquad m^*=\{N,J,(-M)\} \tag{C3}$$

and Σ for the polarization of the gluon

$$\Sigma = 0, \mathscr{L}, \mathscr{M}, \mathscr{E} \tag{C4}$$

The scalar and longitudinal matrix elements, which transmit the instantaneous "Coulomb" interaction between two quarks, are related by current conservation, i.e.,

$$Q^{\mathscr{L}}_{nn'm} = \frac{\varepsilon_{n'} - \varepsilon_n}{\Omega^0_m} Q^0_{nn'm} \tag{C5}$$

We can easily separate the radial and angular integrations, yielding

$$Q^{\Sigma}_{nn'm} = R^{-3/2} R^{\Sigma}_{nn'm} \int \chi^{\mu^+}_{\kappa}(\hat{\mathbf{r}}) Y_{JM}(\hat{\mathbf{r}}) \chi^{\mu'}_{\kappa'}(\hat{\mathbf{r}}) \, d\Omega; \qquad \Sigma = 0, \mathscr{L}, \mathscr{E} \tag{C6}$$

$$Q^{\mathscr{M}}_{nn'm} = R^{-3/2} R^{\mathscr{M}}_{nn'm} \int \chi^{\mu^+}_{\kappa}(\hat{\mathbf{r}}) Y_{JM}(\hat{\mathbf{r}}) \chi^{\mu'}_{-\kappa'}(\hat{\mathbf{r}}) \, d\Omega \tag{C7}$$

where the dependence on the magnetic quantum numbers is now contained in the angular matrix element. Integrating over the angular variables and using the abbreviation $\hat{J} = (2J+1)^{1/2}$ one arrives in terms of $3j$-symbols, at

$$\int \chi^{\mu^+}_{\kappa}(\hat{\mathbf{r}}) Y_{JM}(\hat{\mathbf{r}}) \chi^{\mu'}_{\kappa'}(\hat{\mathbf{r}}) \, d\Omega$$
$$= (-1)^{\mu+1/2} \frac{\hat{j}\hat{J}\hat{j}'}{(4\pi)^{1/2}} \begin{pmatrix} j & J & j' \\ -\mu & M & \mu' \end{pmatrix} \begin{pmatrix} j & J & j' \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{pmatrix} \frac{(-1)^{l+J+l'}+1}{2} \tag{C8}$$

which governs the angular momentum and parity selection rules. After some straightforward but tedious algebra, we obtain for the scalar and longitudinal radial matrix elements (Viollier *et al.*, 1983)

$$R^0_{nn'm} = -\mathscr{N}^0_m \int_0^R j_J(\Omega^0_m r) S_{nn'}(r) r^2 \, dr \tag{C9}$$

$$R^{\mathscr{L}}_{nn'm} = \frac{-\mathscr{N}^0_m}{\Omega^0_m} \int_0^R \{[\Omega^0_m r j_{J+1}(\Omega^0_m r) - J j_J(\Omega^0_m r)] U_{nn'}(r)$$
$$+ (\kappa - \kappa') j_J(\Omega^0_m r) T_{nn'}(r)\} r \, dr \tag{C10}$$

The transverse magnetic and electric radial matrix elements turn out to be of the form

$$R^{\mathscr{M}}_{nn'm} = \frac{\kappa' + \kappa}{[J(J+1)]^{1/2}} \mathscr{N}^{\mathscr{M}}_m \int_0^R j_J(\Omega^{\mathscr{M}}_m r) T_{nn'}(r) r^2 \, dr \tag{C11}$$

$$R^{\mathscr{E}}_{nn'm} = \frac{\mathscr{N}^{\mathscr{E}}_m}{[J(J+1)]^{1/2} \Omega^{\mathscr{E}}_m} \int_0^R \{J(J+1) j_J(\Omega^{\mathscr{E}}_m r) U_{nn'}(r)$$
$$+ (\kappa - \kappa')[J j_J(\Omega^{\mathscr{E}}_m r) - \Omega^{\mathscr{E}}_m r j_{J-1}(\Omega^{\mathscr{E}}_m r)] T_{nn'}(r)\} r \, dr \tag{C12}$$

Here we have introduced the radial functions

$$\begin{aligned} S_{nn'} &= g_n g_{n'} + f_n f_{n'} \\ T_{nn'} &= g_n f_{n'} + f_n g_{n'} \\ U_{nn'} &= g_n f_{n'} - f_n g_{n'} \end{aligned} \tag{C13}$$

which are given in terms of the radial functions of the quarks in the initial and final states.

C2. The Ghost–Gluon Vertex

The vertex integrals that describe the absorption (or emission) of a gluon by a ghost (Figure 1b) are given by integrals of the type

$$T_{mm'm''} = i \int a^0_m(\mathbf{r}) a^0_{m'}(\mathbf{r}) a^0_{m''}(\mathbf{r})\, d^3r \tag{C14}$$

and

$$T^{\Sigma\Sigma'}_{mm'm''} = -i \int \mathbf{a}^{\Sigma}_m(\mathbf{r}) \cdot \mathbf{a}^{\Sigma'}_{m'}(\mathbf{r}) a^0_{m''}(\mathbf{r})\, d^3r \tag{C15}$$

where m, m', and m'' stand for the gluon and ghost quantum numbers, e.g.,

$$m = \{N, J, (M)\} \tag{C16}$$

The integral (C14) can be separated into a radial and an angular part, giving

$$\begin{aligned} T_{mm'm''} = {} & \frac{\mathcal{N}^0_m \mathcal{N}^0_{m'} \mathcal{N}^0_{m''}}{R^{9/2}} R_{JJ'J''}(\Omega^0_m, \Omega^0_{m'}, \Omega^0_{m''}) \\ & \times \int Y_{JM}(\hat{\mathbf{r}}) Y_{J'M'}(\hat{\mathbf{r}}) Y_{J''M''}(\hat{\mathbf{r}})\, d\Omega \end{aligned} \tag{C17}$$

The radial integrals are defined by

$$R_{JJ'J''}(\Omega, \Omega', \Omega'') = \int_0^R j_J(\Omega r) j_{J'}(\Omega' r) j_{J''}(\Omega'' r) r^2\, dr \tag{C18}$$

and the angular integrating yields

$$\begin{aligned} & \int d\Omega\, Y_{J'M'}(\hat{r}) Y_{J''M''}(\hat{r}) Y_{JM}(\hat{r}) \\ & = \begin{pmatrix} J' & J'' & J \\ M' & M'' & M \end{pmatrix} \begin{pmatrix} J' & J'' & J \\ 0 & 0 & 0 \end{pmatrix} \frac{\hat{J}'\hat{J}''\hat{J}}{(4\pi)^{1/2}} \end{aligned} \tag{C19}$$

Separating the radial and angular integrations in the vertex function (C15), we obtain

$$T^{\Sigma\Sigma'}_{mm'm''}=\frac{\mathcal{N}^{\Sigma}_{m}\mathcal{N}^{\Sigma'}_{m'}\mathcal{N}^{0}_{m''}}{\hat{J}\hat{J}'R^{9/2}}\sum_{LL'}\alpha^{\Sigma}_{JL}\alpha^{\Sigma'}_{J'L'}$$
$$\times R_{LL'J}(\Omega^{\Sigma}_{m},\Omega^{\Sigma'}_{m'},\Omega^{0}_{m''})\int \mathbf{Y}_{JLM}(\hat{\mathbf{r}})\cdot\mathbf{Y}_{J'L'M'}(\hat{\mathbf{r}})\,Y_{J''M''}(\hat{\mathbf{r}})\,d\Omega \qquad \text{(C20)}$$

The angular integral is given in terms of $3j$- and $6j$-symbols as

$$\int \mathbf{Y}_{JLM}(\hat{\mathbf{r}})\cdot\mathbf{Y}_{J'L'M'}(\hat{\mathbf{r}})\mathbf{Y}_{J''M''}(\hat{\mathbf{r}})\,d\Omega$$
$$=(-1)^{L+J}\frac{\hat{J}\hat{J}'\hat{J}''\hat{L}\hat{L}'}{(4\pi)^{1/2}}\begin{pmatrix} J & J' & J''\\ M & M' & M''\end{pmatrix}$$
$$\times\begin{pmatrix} L & J'' & L'\\ 0 & 0 & 0\end{pmatrix}\begin{Bmatrix} J' & J'' & J\\ L & 1 & L'\end{Bmatrix} \qquad \text{(C21)}$$

C3. The Three-Gluon Vertex

We now turn to the evaluation of the three-gluon-vertex integral, which describes the absorption (or emission) of a gluon by another gluon (Figure 1c). These integrals can be formulated in terms of the $T_{mm'm''}$ and $T^{\Sigma\Sigma'}_{mm'm''}$, which have been defined previously, and an additional integral of the type

$$T^{\Sigma\Sigma'\Sigma''}_{mm'm''}=\int [\mathbf{a}^{\Sigma}_{m}(\mathbf{r})\times\mathbf{a}^{\Sigma'}_{m'}(\mathbf{r})]\cdot[\nabla\times\mathbf{a}^{\Sigma''}_{m''}(\mathbf{r})]\,d^3r \qquad \text{(C22)}$$

Separating the integration into a radial and an angular part, we arrive at

$$T^{\Sigma\Sigma'\Sigma''}_{mm'm''}=\frac{\Omega^{\Sigma''}_{m''}\mathcal{N}^{\Sigma}_{m}\mathcal{N}^{\Sigma'}_{m'}\mathcal{N}^{\Sigma''}_{m''}}{JJ'J''R^{11/2}}\sum_{LL'L''}\alpha^{\Sigma}_{JL}\alpha^{\Sigma'}_{J'L'}\beta^{\Sigma''}_{J''L''}R_{LL'L''}(\Omega^{\Sigma}_{m},\Omega^{\Sigma'}_{m'},\Omega^{\Sigma''}_{m''})$$
$$\times(-i)\int [\mathbf{Y}_{JLM}(\hat{\mathbf{r}})\times\mathbf{Y}_{J'L'M'}(\hat{\mathbf{r}})]\cdot\mathbf{Y}_{J''L''M''}(\hat{\mathbf{r}})\,d\Omega \qquad \text{(C23)}$$

where the angular integral is given in terms of $3j$- and $9j$-symbols by

$$(-i)\int [\mathbf{Y}_{JLM}(\hat{\mathbf{r}})\times\mathbf{Y}_{J'L'M'}(\hat{\mathbf{r}})]\cdot\mathbf{Y}_{J''L''M''}(\hat{\mathbf{r}})\,d\Omega$$
$$=\left(\frac{3}{2\pi}\right)^{1/2}\hat{J}\hat{J}'\hat{J}''\hat{L}\hat{L}'\hat{L}''\begin{pmatrix} L & L' & L''\\ 0 & 0 & 0\end{pmatrix}$$
$$\times\begin{Bmatrix} J & J' & J''\\ 1 & 1 & 1\\ L & L' & L''\end{Bmatrix}\begin{pmatrix} J & J' & J''\\ M & M' & M''\end{pmatrix} \qquad \text{(C24)}$$

C4. The Four-Gluon Vertex

The four-gluon vertex, which describes the elementary interaction of four gluons (Figure 1d), is given by integrals of the type

$$F^{\Sigma\Sigma'}_{mm'm''m'''} = \int \mathbf{a}^{\Sigma}_{m}(\mathbf{r}) \cdot \mathbf{a}^{\Sigma'}_{m'}(\mathbf{r}) a^{0}_{m''}(\mathbf{r}) a^{0}_{m'''}(\mathbf{r})\, d^3r \tag{C25}$$

and

$$F^{\Sigma\Sigma'\Sigma''\Sigma'''}_{mm'm''m'''} = \int \mathbf{a}^{\Sigma}_{m}(\mathbf{r}) \cdot \mathbf{a}^{\Sigma'}_{m'}(\mathbf{r}) \mathbf{a}^{\Sigma''}_{m''}(\mathbf{r}) \cdot \mathbf{a}^{\Sigma'''}_{m'''}(\mathbf{r})\, d^3r \tag{C26}$$

For the first integral (C26) we obtain

$$\begin{aligned} F^{\Sigma\Sigma'}_{mm'm''m'''} = {} & \frac{\mathcal{N}^{\Sigma}_{m}\mathcal{N}^{\Sigma}_{m'}\mathcal{N}^{0}_{m''}\mathcal{N}^{0}_{m'''}}{\hat{J}\hat{J}' R^6} \\ & \times \sum_{LL'} \alpha^{\Sigma}_{JL}\alpha^{\Sigma'}_{J'L'} R_{JJ'J''J'''}(\Omega^{\Sigma}_{m},\ \Omega^{\Sigma'}_{m'}, \Omega^{0}_{m''}, \Omega^{0}_{m'''}) \\ & \times \int \mathbf{Y}_{JLM}(\hat{\mathbf{r}}) \cdot \mathbf{Y}_{J'L'M'}(\hat{\mathbf{r}})\, Y_{J''M''}(\hat{\mathbf{r}})\, Y_{J'''M'''}(\hat{\mathbf{r}})\, d\Omega \end{aligned} \tag{C27}$$

Here the radial integral is defined as

$$R_{JJ'J''J'''}(\Omega, \Omega', \Omega'', \Omega''') = \int_0^R j_J(\Omega r) j_{J'}(\Omega' r) j_{J''}(\Omega'' r) j_{J'''}(\Omega''' r) r^2\, dr \tag{C28}$$

and the angular integration yields

$$\begin{aligned} & \int \mathbf{Y}_{JLM}(\hat{\mathbf{r}}) \cdot \mathbf{Y}_{J'L'M'}(\hat{\mathbf{r}})\, Y_{J''M''}(\hat{\mathbf{r}})\, Y_{J'''M'''}(\hat{\mathbf{r}})\, d\Omega \\ & \quad = \sum_{k\kappa} \begin{pmatrix} J & J' & k \\ M & M' & \kappa \end{pmatrix} \begin{pmatrix} J'' & J''' & k \\ M'' & M''' & -\kappa \end{pmatrix} \frac{(-1)^{k+\kappa+J+L'}}{4\pi} \\ & \quad \times (2k+1)\hat{J}\hat{J}'(2J''+1)(2J'''+1)\hat{L}\hat{L}' \\ & \quad \times \begin{pmatrix} J'' & J''' & k \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} L & L' & k \\ 0 & 0 & 0 \end{pmatrix} \begin{Bmatrix} J & J' & k \\ L' & L & 1 \end{Bmatrix} \end{aligned} \tag{C29}$$

The second integral (C26) also separates into a radial and an angular part, yielding

$$\begin{aligned} F^{\Sigma\Sigma'\Sigma''\Sigma'''}_{mm'm''m'''} = {} & \frac{\mathcal{N}^{\Sigma}_{m}\mathcal{N}^{\Sigma'}_{m'}\mathcal{N}^{\Sigma''}_{m''}\mathcal{N}^{\Sigma'''}_{m'''}}{\hat{J}\hat{J}'\hat{J}''\hat{J}''' R^6} \sum_{LL'L''L'''} \alpha^{\Sigma}_{JL}\alpha^{\Sigma'}_{J'L'}\alpha^{\Sigma''}_{J''L''}\alpha^{\Sigma'''}_{J'''L'''} \\ & \times R_{JJ'J''J'''}(\Omega^{\Sigma}_{m}, \Omega^{\Sigma'}_{m'}, \Omega^{\Sigma''}_{m''}, \Omega^{\Sigma'''}_{m'''}) \\ & \times \int \mathbf{Y}_{JLM}(\hat{\mathbf{r}}) \cdot \mathbf{Y}_{J'L'M'}(\hat{\mathbf{r}}) \mathbf{Y}_{J''L''M''}(\hat{r}) \cdot \mathbf{Y}_{J'''L'''M'''}(\hat{\mathbf{r}})\, d\Omega \end{aligned} \tag{C30}$$

Here one obtains for the angular integration

$$\int \mathbf{Y}_{JLM}(\hat{\mathbf{r}}) \cdot \mathbf{Y}_{J'L'M'}(\hat{\mathbf{r}}) \mathbf{Y}_{J''L''M''}(\hat{\mathbf{r}}) \cdot \mathbf{Y}_{J'''L'''M'''}(\hat{\mathbf{r}})\, d\Omega$$
$$= \sum_k \begin{pmatrix} J & J' & k \\ M & M' & \kappa \end{pmatrix} \begin{pmatrix} J'' & J''' & k \\ M'' & M''' & -\kappa \end{pmatrix} \frac{(-1)^{\kappa+J+L'+J''+L'''}}{4\pi}$$
$$\times (2k+1)\hat{J}\hat{J}'\hat{J}''\hat{J}'''\hat{L}\hat{L}'\hat{L}''\hat{L}'''$$
$$\times \begin{pmatrix} L & L' & k \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} L'' & L''' & k \\ 0 & 0 & 0 \end{pmatrix} \begin{Bmatrix} J & J' & k \\ L' & L & 1 \end{Bmatrix} \begin{Bmatrix} J'' & J''' & k \\ L''' & L'' & 1 \end{Bmatrix} \tag{C31}$$

APPENDIX D. INTERACTING TWO-PARTICLE SYSTEMS

D1. Systems Consisting of Quarks and Antiquarks

A system consisting of two quarks, two antiquarks, or a quark–antiquark pair that interact through the exchange of a gluon can be represented by the Feynman diagrams in Figures 2a, 2b, or 2c, respectively. The energy shifts are obtained from equations (5.1) or (5.3), where $|\hat{\Phi}_k\rangle$ denotes a two-particle eigenstate of the noninteracting Hamiltonian $:\hat{H}_0:$, i.e.,

$$|\hat{\Phi}_k\rangle = \hat{a}^+_{c_1n_1}\hat{a}^+_{c_2n_2}|\hat{0}\rangle \tag{D1a}$$

$$|\hat{\Phi}_k\rangle = \hat{b}^+_{c_1n_1}\hat{b}^+_{c_2n_2}|\hat{0}\rangle \tag{D1b}$$

$$|\hat{\Phi}_k\rangle = \hat{a}^+_{c_1n_1}\hat{b}^+_{c_2n_2}|\hat{0}\rangle \tag{D1c}$$

After a straightforward calculation we arrive at the two-body operators

$$\hat{V} = g^2\left(\frac{\lambda_a}{2}\right)_{c'c}\left(\frac{\lambda_a}{2}\right)_{d'd}\delta_{f'f}\delta_{g'g}$$
$$\times \frac{g^{\Sigma\Sigma}}{2\Omega^{\Sigma}_m}\frac{Q^{\Sigma}_{n'nm}\tilde{Q}^{\Sigma}_{p'pm}}{\Omega^{\Sigma}_m+\varepsilon_{p'}-\varepsilon_p}\hat{a}^+_{c'n'}\hat{a}^+_{d'p'}\hat{a}_{cn}\hat{a}_{dp} \tag{D2a}$$

$$\hat{V} = g^2\left(-\frac{\lambda_a}{2}\right)_{cc'}\left(-\frac{\lambda_a}{2}\right)_{dd'}\delta_{f'f}\delta_{g'g}$$
$$\times \frac{g^{\Sigma\Sigma}}{2\Omega^{\Sigma}_m}\frac{Q^{\Sigma}_{-n-n'm}\tilde{Q}^{\Sigma}_{-p-p'm}}{\Omega^{\Sigma}_m+\varepsilon_{p'}-\varepsilon_p}\hat{b}^+_{c'n'}\hat{b}^+_{d'p'}\hat{b}_{cn}\hat{b}_{dp} \tag{D2b}$$

$$\hat{V} = g^2\left(\frac{\lambda_a}{2}\right)_{c'c}\left(-\frac{\lambda_a}{2}\right)_{dd'}\delta_{f'f}\delta_{g'g}$$
$$\times \frac{g^{\Sigma\Sigma}}{2\Omega^{\Sigma}_m}Q^{\Sigma}_{n'nm}\tilde{Q}^{\Sigma}_{-p-p'm}\left(\frac{1}{\Omega^{\Sigma}_m+\varepsilon_{p'}-\varepsilon_p}+\frac{1}{\Omega^{\Sigma}_m+\varepsilon_{n'}-\varepsilon_n}\right)\hat{a}^+_{c'n'}\hat{b}^+_{d'p'}\hat{a}_{cn}\hat{b}_{dp} \tag{D2c}$$

As usual, a summation over all indices occurring repeatedly is understood. The two-body operators V_{12} have, for all three cases under consideration, the form

$$V_{12}=\frac{\alpha_s}{R}\mathbf{F}_1\cdot\mathbf{F}_2\sum_{J=0}^{\infty}\mu_{12}(J)K_{12}(J) \tag{D3}$$

where α_s is the "fine structure constant" of the strong interaction

$$\alpha_s=g^2/4\pi \tag{D4}$$

and R is the radius of the cavity. Here, the vector $\mathbf{F}$ stands for the eight generators of $SU(3)_{\text{color}}$ in the appropriate representation, i.e.,

$$\mathbf{F}=\begin{cases}\frac{1}{2}\boldsymbol{\lambda} & \text{for quarks}\\ -\frac{1}{2}\boldsymbol{\lambda}^T & \text{for antiquarks}\end{cases} \tag{D5}$$

The two-body operators $K_{12}(J)$ determine the angular momentum structure of the interaction V_{12}. Their matrix elements are most easily given in terms of the function $F_{JM}(n, n')$,

$$F_{JM}(n,n')=(-1)^{\mu+1/2}\hat{j}\hat{J}\hat{j}'\begin{pmatrix} j & J & j'\\ \frac{1}{2} & 0 & -\frac{1}{2}\end{pmatrix}\begin{pmatrix} j & J & j'\\ -\mu & M & \mu'\end{pmatrix} \tag{D6}$$

This is, up to the parity selection rule and a factor of $(4\pi)^{-1/2}$, the angular integral contained in the vertex function $Q^{\Sigma}_{nn'm}$ [equation (C1)]. Now, we have

$$\langle n_1', n_2'|K_{12}(J)|n_1, n_2\rangle=(2J+1)\sum_{M=-J}^{J}(-1)^M F_{JM}(n_1', n_1)F_{J-M}(n_2', n_2) \tag{D7a}$$

$$\langle \bar{n}_1', \bar{n}_2'|K_{12}(J)|\bar{n}_1, \bar{n}_2\rangle=(2J+1)\sum_{M=-J}^{J}(-1)^M F_{JM}(-\bar{n}_1, -\bar{n}_1')F_{J-M}(-\bar{n}_2, -\bar{n}_2') \tag{D7b}$$

$$\langle n_1', \bar{n}_2'|K_{12}(J)|n_1, \bar{n}_2\rangle=(2J+1)\sum_{M=-J}^{J}(-1)^M F_{JM}(n_1', n_1)F_{J-M}(-\bar{n}_2, -\bar{n}_2') \tag{D7c}$$

for the quark–quark, antiquark–antiquark, and quark–antiquark interactions, respectively. As an example, the ket $|n, \bar{n}\rangle$ describes here the direct product state built up of a quark with quantum numbers n and an antiquark with quantum numbers $\bar{n}$. The operators $\mu_{12}(J)$ originate in the radial integral in the vertex function $Q^{\Sigma}_{nn'm}$ [equation (C1)]. In addition, they carry the parity selection rule, which we have omitted in the definition of $K_{12}(J)$.

With the help of the modified radial integrals

$$S^{\Sigma}_{nn'm} = \frac{1-\eta_{\Sigma} g^{\Sigma\Sigma}(-1)^{l+J+l'}}{2} R^{\Sigma}_{nn'm} \tag{D8}$$

the matrix elements of $\mu_{12}(J)$ are expressed as follows:

$$\langle n_1', n_2'|\mu_{12}(J)|n_1, n_2\rangle = \sum_{N\Sigma} \frac{\eta_{\Sigma} g^{\Sigma\Sigma}}{2(2J+1)} S^{\Sigma}_{n_1'n_1m} S^{\Sigma}_{n_2'n_2m} \times \frac{1}{R^2\Omega^{\Sigma}_m}\left(\frac{1}{\Omega^{\Sigma}_m + \varepsilon'_{n_1} - \varepsilon_{n_1}} + \frac{1}{\Omega^{\Sigma}_m + \varepsilon_{n_2'} - \varepsilon_{n_2}}\right) \tag{D9a}$$

$$\langle \bar{n}_1', \bar{n}_2'|\mu_{12}(J)|\bar{n}_1, \bar{n}_2\rangle = \sum_{N\Sigma} \frac{\eta_{\Sigma} g^{\Sigma\Sigma}}{2(2J+1)} S^{\Sigma}_{-\bar{n}_1-\bar{n}_1'm} S^{\Sigma}_{-\bar{n}_2-\bar{n}_2'm} \times \frac{1}{R^2\Omega^{\Sigma}_m}\left(\frac{1}{\Omega^{\Sigma}_m + \varepsilon_{\bar{n}_1'} - \varepsilon_{\bar{n}_1}} + \frac{1}{\Omega^{\Sigma}_m + \varepsilon_{\bar{n}_2'} - \varepsilon_{\bar{n}_2}}\right) \tag{D9b}$$

$$\langle n_1', \bar{n}_2'|\mu_{12}(J)|n_1, \bar{n}_2\rangle = \sum_{N\Sigma} \frac{2}{3}\frac{\eta_{\Sigma} g^{\Sigma\Sigma}}{2J+1} S^{\Sigma}_{n_1'n_1m} S^{\Sigma}_{-\bar{n}_2-\bar{n}_2'm} \times \frac{1}{R^2\Omega^{\Sigma}_m}\left(\frac{1}{\Omega^{\Sigma}_m + \varepsilon_{n_1'} - \varepsilon_{n_1}} + \frac{1}{\Omega^{\Sigma}_m + \varepsilon_{\bar{n}_2'} - \varepsilon_{\bar{n}_2}}\right) \tag{D9c}$$

The quark-antiquark system can interact, in addition to the one-gluon exchange discussed above, through the annihilation into a gluon, which is represented by the Feynman diagram in Fig. 2d. Of course, this interaction is also contained in the part of the interaction Hamiltonian shown in equation (5.5). The corresponding two-body operator is given by

$$\hat{V} = g^2\left(\frac{\lambda_a}{2}\right)_{c'd'}\left(\frac{\lambda_a}{2}\right)_{dc} \delta_{f'g'}\delta_{fg} \frac{g^{\Sigma\Sigma}}{2\Omega^{\Sigma}_m} Q^{\Sigma}_{n'-p'm}\tilde{Q}^{\Sigma}_{-pnm} \times\left(\frac{1}{\Omega^{\Sigma}_m - \varepsilon_p - \varepsilon_n} + \frac{1}{\Omega^{\Sigma}_m + \varepsilon_{p'} + \varepsilon_{n'}}\right)\hat{a}^{+}_{c'n'}\hat{b}^{+}_{d'p'}\hat{a}_{cn}\hat{b}_{dp} \tag{D10}$$

Before writing down the two-body operator in first quantization, we have to cast the color and flavor factors into a suitable form. Using the completeness and trace-orthogonality of $SU(N)$ generators, we derive the following identies

$$\left(\frac{\lambda_a}{2}\right)_{c'd'}\left(\frac{\lambda_a}{2}\right)_{dc} = \tfrac{4}{9}\delta_{d'd}\delta_{c'c} + \frac{1}{3}\left(\frac{\lambda_a}{2}\right)_{c'c}\left(-\frac{\lambda_a}{2}\right)_{dd'} \tag{D11}$$

for $SU(3)_{\text{color}}$ and, assuming $SU(2)_{\text{isospin}}$ for the flavor group,

$$\delta_{f'g'}\delta_{fg} = 2\left[\tfrac{1}{4}\delta_{f'f}\delta_{g'g} - \left(\frac{\tau_l}{2}\right)_{f'f}\left(-\frac{\tau_l}{2}\right)_{gg'}\right] \tag{D12}$$

The last term in equation (D12) contains the $SU(2)_{\text{isospin}}$ generators in the representations that correspond to quarks and antiquarks, respectively

$$\mathbf{T}=\begin{cases}\frac{1}{2}\boldsymbol{\tau} & \text{for quarks}\\ -\frac{1}{2}\boldsymbol{\tau}^T & \text{for antiquarks}\end{cases} \tag{D13}$$

With the help of equations (D11) and (D12), the two-body operator in first quantization takes the form

$$V_{12}=\frac{\alpha_s}{R}[\tfrac{1}{4}-\mathbf{T}_1\cdot\mathbf{T}_2][\mathbf{F}_1\cdot\mathbf{F}_2+\tfrac{4}{3}]\sum_{J=0}^{\infty}\mu_{12}(J)K_{12}(J) \tag{D14}$$

The angular momentum dependence of V_{12} is given by

$$\langle n_1', \bar{n}_2'|K_{12}(J)|n_1, \bar{n}_2\rangle=\tfrac{1}{2}(2J+1)\sum_{M=-J}^{J}(-1)^M F_{JM}(-\bar{n}_2, n_1)F_{J-M}(n_1', -\bar{n}_2') \tag{D15}$$

and $\mu_{12}(J)$ contains the radial integrals and the parity selection rule

$$\langle n_1', \bar{n}_2'|\mu_{12}(J)|n_1, \bar{n}_2\rangle=\sum_{N\Sigma}\frac{2}{3}\frac{n_\Sigma g^{\Sigma\Sigma}}{2J+1}S^{\Sigma}_{-\bar{n}_2 n_1 m}S^{\Sigma}_{n_1'-\bar{n}_2' m}$$
$$\times\frac{1}{R^2\Omega^{\Sigma}_m}\left[\frac{1}{\Omega^{\Sigma}_m-\varepsilon_{n_1}-\varepsilon_{\bar{n}_2}}+\frac{1}{\Omega^{\Sigma}_m+\varepsilon_{n_1'}+\varepsilon_{\bar{n}_2'}}\right] \tag{D16}$$

D2. Systems Consisting of Quarks or Antiquarks and Gluons

The quark-gluon interaction in second-order perturbation theory has two sources in the interaction Hamiltonian. The contribution we discuss first comes again from the part of $H_{\text{int}}(t)$ shown in equation (5.5). It gives rise to the Compton diagrams in Figures 2e, 2f, 2h, and 2i. As opposed to the quark-quark case, the Compton interaction is obtained by contracting two quark fields in the Wick decomposition of the time-ordered product. There are two different, nonvanishing possibilities of contracting two of the four quark fields in equation (5.3). They lead to the direct (Figure 2e or Figure 2h) and the exchange (Figure 2f or Figure 2i) Compton diagrams.

Evaluating the matrix elements $V_{k'k}$, equation (5.3), with the state vectors of the form

$$|\hat{\Phi}_k\rangle=\hat{c}^{\Sigma+}_{a_1 m_1}\hat{a}^{+}_{c_2 n_2}|\hat{0}\rangle \tag{D17a}$$

$$|\hat{\Phi}_k\rangle=\hat{c}^{\Sigma+}_{a_1 m_1}\hat{b}^{+}_{c_2 n_2}|\hat{0}\rangle \tag{D17b}$$

for the quark-gluon or antiquark-gluon systems, respectively, we arrive at the two-body operators

$$\hat{V}=-g^2\left(\frac{\lambda_{a'}}{2}\frac{\lambda_a}{2}\right)_{c'c}\delta_{f'f}\frac{1}{2(\Omega_m^\Sigma\Omega_{m'}^{\Sigma'})^{1/2}}\left(\frac{Q^\Sigma_{-pnm}\tilde{Q}^{\Sigma'}_{n'-pm'}}{\varepsilon_p+\varepsilon_{n'}+\Omega^{\Sigma'}_{m'}}-\frac{Q^\Sigma_{pnm}\tilde{Q}^{\Sigma'}_{n'pm'}}{\varepsilon_p-\varepsilon_n-\Omega^\Sigma_m}\right)$$
$$\times\,\hat{c}^{\Sigma'+}_{a'm'}\hat{a}^+_{c'n'}\hat{c}^\Sigma_{am}\hat{a}_{cn} \tag{D18a}$$

$$\hat{V}=g^2\left(\frac{\lambda_a}{2}\frac{\lambda_{a'}}{2}\right)_{cc'}\delta_{f'f}\frac{1}{2(\Omega_m^\Sigma\Omega_{m'}^{\Sigma'})^{1/2}}\left(\frac{Q^\Sigma_{-n-pm}\tilde{Q}^{\Sigma'}_{-p-n'-m'}}{\varepsilon_p-\varepsilon_n-\Omega^\Sigma_m}-\frac{Q^\Sigma_{-npm}\tilde{Q}^{\Sigma'}_{p-n'm'}}{\varepsilon_p+\varepsilon_{n'}+\Omega^{\Sigma'}_{m'}}\right)$$
$$\times\,\hat{c}^{\Sigma'+}_{a'm'}\hat{b}^+_{c'n'}\hat{c}^\Sigma_{am}\hat{b}_{cn} \tag{D18b}$$

for the direct diagrams in Figures 2e and 2h, and

$$\hat{V}=-g^2\left(\frac{\lambda_a}{2}\frac{\lambda_{a'}}{2}\right)_{c'c}\delta_{f'f}\frac{1}{2(\Omega_m^\Sigma\Omega_{m'}^{\Sigma'})^{1/2}}\left(\frac{Q^\Sigma_{n'-pm}\tilde{Q}^{\Sigma'}_{-pnm'}}{\varepsilon_p+\varepsilon_{n'}-\Omega^\Sigma_m}-\frac{Q^\Sigma_{n'pm}\tilde{Q}^{\Sigma'}_{pnm'}}{\varepsilon_p-\varepsilon_n+\Omega^{\Sigma'}_{m'}}\right)$$
$$\times\,\tilde{c}^{\Sigma'+}_{a'm'}\hat{a}^+_{c'n'}\hat{c}^\Sigma_{am}\hat{a}_{cn} \tag{D19a}$$

$$\hat{V}=g^2\left(\frac{\lambda_{a'}}{2}\frac{\lambda_a}{2}\right)_{cc'}\delta_{f'f}\frac{1}{2(\Omega_m^\Sigma\Omega_{m'}^{\Sigma'})^{1/2}}\left(\frac{Q^\Sigma_{-p-n'm}\tilde{Q}^{\Sigma'}_{-n-pm'}}{\varepsilon_p-\varepsilon_n+\Omega^{\Sigma'}_{m'}}-\frac{Q^\Sigma_{p-n'm}\tilde{Q}^{\Sigma'}_{-npm'}}{\varepsilon_p+\varepsilon_{n'}-\Omega^\Sigma_m}\right)$$
$$\times\,\hat{c}^{\Sigma'+}_{a'm'}\hat{b}^+_{c'n'}\hat{c}^\Sigma_{am}\hat{b}_{cn} \tag{D19b}$$

for the exchange Compton diagrams as shown in Figures 2f and 2i. In order to define the two-body operators V_{12} in first quantization, we still have to rearrange the color factors in equations (D18) and (D19). This is achieved with the help of the identity

$$\frac{\lambda_a}{2}\frac{\lambda_b}{2}=\frac{1}{6}\left(4\frac{\lambda_c}{2}\frac{\lambda_d}{2}f_{aec}f_{bed}-\delta_{ab}\mathbb{1}\right) \tag{D20}$$

Now, we easily arrive at

$$V_{12}=\frac{\alpha_s}{18R}[4(\mathbf{F}_1\cdot\mathbf{F}_2)^2-1]\sum_{p=1}^{\infty}\mu^{\mathrm{D}}_{12}(p)K^{\mathrm{D}}_{12}(p) \tag{D21}$$

for the direct diagrams and

$$V_{12}=\frac{\alpha_s}{18R}[4(\mathbf{F}_1\cdot\mathbf{F}_2)^2-6\mathbf{F}_1\cdot\mathbf{F}_2-1]\sum_{p=1}^{\infty}\mu^{\mathrm{E}}_{12}(p)K^{\mathrm{E}}_{12}(p) \tag{D22}$$

for the exchange diagrams. Equations (D21) and (D22) are correct for both the quark-gluon and the antiquark-gluon systems. Here, one of the generators in the product $\mathbf{F}_1\cdot\mathbf{F}_2$ acts on the quark or antiquark [see equation (D5)] and the other on the gluon,

$$(F_a)_{bc}=-if_{abc}\qquad\text{for gluons} \tag{D23}$$

The angular momentum structure of the operator V_{12} is again contained in the two-body operators $K^{\mathrm{D}}_{12}(p)$ and $K^{\mathrm{E}}_{12}(p)$,

$$\langle \Sigma_1' m_1', n_2' | K^{\mathrm{D}}_{12}(p) | \Sigma_1 m_1, n_2 \rangle = \frac{6}{4} \sum_{\substack{\mu \\ \kappa = p}} F_{J_1 M_1}(n, n_2) F_{J_1' M_1'}(n, n_2') \tag{D23a}$$

$$\langle \Sigma_1' m_1', \bar{n}_2' | K^{\mathrm{D}}_{12}(p) | \Sigma_1 m_1, \bar{n}_2 \rangle = \frac{6}{4} \sum_{\substack{\mu \\ \kappa = p}} F_{J_1 M_1}(-\bar{n}_2, n) F_{J_1' M_1'}(-\bar{n}_2', n) \tag{D23b}$$

$$\langle \Sigma_1' m_1', n_2' | K^{\mathrm{E}}_{12}(p) | \Sigma_1 m_1, n_2 \rangle = \frac{6}{4} \sum_{\substack{\mu \\ \kappa = p}} F_{J_1 M_1}(n_2', n) F_{J_1' M_1'}(n_2, n) \tag{D24a}$$

$$\langle \Sigma_1' m_1', \bar{n}_2' | K^{\mathrm{E}}_{12}(p) | \Sigma_1 m_1, \bar{n}_2 \rangle = \frac{6}{4} \sum_{\substack{\mu \\ \kappa = p}} F_{J_1 M_1}(n_2, -\bar{n}_2') F_{J_1' M_1'}(n, -\bar{n}_2) \tag{D24b}$$

The two-body operators $\mu^{\mathrm{D}}_{12}(p)$ and $\mu^{\mathrm{E}}_{12}(p)$, which contain the radial integral of $Q^{\Sigma}_{nn'm}$, are given in the various cases by

$$\langle \Sigma_1' m_1', n_2' | \mu^{\mathrm{D}}_{12}(p) | \Sigma_1 m_1, n_2 \rangle = \sum_{\substack{\mu \\ \kappa = \pm p}} \frac{\eta_{\Sigma_1'}}{R^2 (\Omega^{\Sigma_1}_{m_1} \Omega^{\Sigma_1'}_{m_1'})^{1/2}} \times \left(\frac{S^{\Sigma_1'}_{n_2' n m_1'} S^{\Sigma_1}_{n n_2 m_1}}{\varepsilon_n - \varepsilon_{n_2} - \Omega^{\Sigma_1}_{m_1}} - \frac{S^{\Sigma_1'}_{n_2' - n m_1'} S^{\Sigma_1}_{-n n_2 m_1}}{\varepsilon_n + \varepsilon_{n_2'} + \Omega^{\Sigma_1'}_{m_1'}} \right) \tag{D25a}$$

$$\langle \Sigma_1' m_1', \bar{n}_2' | \mu^{\mathrm{D}}_{12}(p) | \Sigma_1 m_1, \bar{n}_2 \rangle = \sum_{\substack{\mu \\ \kappa = \pm p}} \frac{\eta_{\Sigma_1'}}{R^2 (\Omega^{\Sigma_1}_{m_1} \Omega^{\Sigma_1'}_{m_1'})^{1/2}} \times \frac{S^{\Sigma_1}_{-\bar{n}_2 - n m_1} S^{\Sigma_1'}_{-n - \bar{n}_2' m_1'}}{(\varepsilon_n - \varepsilon_{\bar{n}_2} - \Omega^{\Sigma_1}_{m_1}} - \frac{S^{\Sigma_1}_{-\bar{n}_2 n m_1} S^{\Sigma_1'}_{n - \bar{n}_2' m_1'}}{\varepsilon_n + \varepsilon_{\bar{n}_2'} + \Omega^{\Sigma_1'}_{m_1'}} \Bigg) \tag{D25b}$$

$$\langle \Sigma_1' m_1', n_2' | \mu^{\mathrm{E}}_{12}(p) | \Sigma_1 m_1, n_2 \rangle = \sum_{\substack{\mu \\ \kappa = \pm p}} \frac{\eta_{\Sigma_1'}}{R^2 (\Omega^{\Sigma_1}_{m_1} \Omega^{\Sigma_1'}_{m_1'})^{1/2}} \times \left(\frac{S^{\Sigma_1}_{n_2' n m_1} S^{\Sigma_1'}_{n n_2 m_1'}}{\varepsilon_n - \varepsilon_{n_2} + \Omega^{\Sigma_1'}_{m_1'}} - \frac{S^{\Sigma_1}_{n_2' - n m_1} S^{\Sigma_1'}_{-n n_2 m_1'}}{\varepsilon_n + \varepsilon_{n_2'} - \Omega^{\Sigma_1}_{m_1}} \right) \tag{D26a}$$

$$\langle \Sigma_1' m_1', \bar{n}_2' | \mu^{\mathrm{E}}_{12}(p) | \Sigma_1 m_1, \bar{n}_2 \rangle = \sum_{\substack{\mu \\ \kappa = \pm p}} \frac{\eta_{\Sigma_1'}}{R^2 (\Omega^{\Sigma_1}_{m_1} \Omega^{\Sigma_1'}_{m_1'})^{1/2}} \times \left(\frac{S^{\Sigma_1}_{-n - \bar{n}_2' m_1} S^{\Sigma_1'}_{-\bar{n}_2 - n m_1'}}{\varepsilon_n - \varepsilon_{\bar{n}_2} + \Omega^{\Sigma_1'}_{m_1'}} - \frac{S^{\Sigma_1}_{n - \bar{n}_2' m_1} S^{\Sigma_1'}_{-\bar{n}_2 n m_1'}}{\varepsilon_n + \varepsilon_{\bar{n}_2'} - \Omega^{\Sigma_1}_{m_1}} \right) \tag{D26b}$$

Throughout equations (D23)-(D26), the cases (a) and (b) correspond to the quark-gluon and antiquark-gluon systems, respectively. The ket $|\Sigma m, n\rangle$

denotes the direct product state consisting of a gluon with polarization Σ and quantum numbers m and a quark with quantum numbers n.

We now turn to the second contribution to the quark–gluon interaction, which is transmitted via the exchange of a virtual gluon and is depicted in Figures 2i and 2j. This process cannot take place in a theory based on an Abelian gauge group (e.g., quantum electrodynamics), where the structure constants f_{abc} vanish.

Before we evaluate the two-body operator $\hat{V}$, it is convenient to introduce the functions $U^{\Sigma_1\Sigma_2\Sigma_3}_{m_1m_2m_3}(\sigma_1, \sigma_3)$ that describe the three-gluon vertex. They are given by

$$\begin{pmatrix} J_1 & J_2 & J_3 \\ M_1 & M_2 & M_3 \end{pmatrix} U^{\Sigma_1\Sigma_2\Sigma_3}_{m_1m_2m_3}(\sigma_1, \sigma_2) = (4\pi)^{1/2} R^{5/2} (T^{\Sigma_1\Sigma_2\Sigma_3}_{m_1m_2m_3} + T^{\Sigma_3\Sigma_1\Sigma_2}_{m_3m_1m_2} + T^{\Sigma_2\Sigma_3\Sigma_1}_{m_2m_3m_1}) \tag{D27}$$

if none of the polarizations Σ_1, Σ_2, and Σ_3 is scalar, and

$$\begin{pmatrix} J_1 & J_2 & J_3 \\ M_1 & M_2 & M_3 \end{pmatrix} U^{\Sigma_1\Sigma_2 0}_{m_1m_2m_3}(\sigma_1, \sigma_2) = (4\pi)^{1/2} R^{5/2} (\sigma_2 \Omega^{\Sigma_2}_{m_2} - \sigma_1 \Omega^{\Sigma_1}_{m_1}) T^{\Sigma_1\Sigma_2}_{m_1m_2m_3} \tag{D28}$$

if one of the gluons is scalar. Due to the restriction to physical external states, no other combination of gluon polarizations can occur at the three-gluon vertex in second-order perturbation theory. The parameters σ_1 and σ_2 can take the values $+1$ or -1. Obviously, they are not necessary in equation (D27); however, it is convenient to have the same notation for both equations (D27) and (D28). The integrals $T^{\Sigma_1\Sigma_2}_{m_1m_2m_3}$ and $T^{\Sigma_1\Sigma_2\Sigma_3}_{m_1m_2m_3}$ are defined in Appendix C, equations (C15) and (C22), respectively.

The two-body operators describing the one-gluon exchange between a quark or an antiquark and a gluon are easily found to be

$$\begin{aligned} \hat{V} = &-g^2 \left(\frac{\lambda_b}{2}\right)_{c'c} (-if_{ba'a}) \delta_{f'f} \frac{\eta_{\Sigma'}\eta_{\Sigma''} g^{\Sigma''\Sigma''}}{4\Omega^{\Sigma''}_{m''}(\Omega^{\Sigma}_{m}\Omega^{\Sigma'}_{m'})^{1/2}} \frac{(-1)^{M'+M''}}{(4\pi)^{1/2} R^{5/2}} \\ &\times \begin{pmatrix} J & J' & J'' \\ M & -M' & -M'' \end{pmatrix} Q^{\Sigma''}_{n'nm''} U^{\Sigma\Sigma'\Sigma''}_{mm'm''}(-, +) \\ &\times \left(\frac{1}{\Omega^{\Sigma''}_{m''} + \Omega^{\Sigma'}_{m'} - \Omega^{\Sigma}_{m}} + \frac{1}{\Omega^{\Sigma''}_{m''} + \varepsilon_{n'} - \varepsilon_n} \right) \hat{c}^{\Sigma'+}_{a'm'} \hat{a}^{+}_{c'n'} \hat{c}^{\Sigma}_{am} \hat{a}_{cn} \end{aligned} \tag{D29a}$$

$$\begin{aligned} \hat{V} = &-g^2 \left(-\frac{\lambda_b}{2}\right)_{cc'} (-if_{ba'a}) \delta_{f'f} \frac{\eta_{\Sigma'}\eta_{\Sigma''} g^{\Sigma''\Sigma''}}{4\Omega^{\Sigma''}_{m''}(\Omega^{\Sigma}_{m}\Omega^{\Sigma'}_{m'})^{1/2}} \frac{(-1)^{M'+M''}}{(4\pi)^{1/2} R^{5/2}} \\ &\times \begin{pmatrix} J & J' & J'' \\ M & -M' & -M'' \end{pmatrix} Q^{\Sigma''}_{-n-n'm''} U^{\Sigma\Sigma'\Sigma''}_{mm'm''}(-, +) \\ &\times \left(\frac{1}{\Omega^{\Sigma''}_{m''} + \Omega^{\Sigma'}_{m'} - \Omega^{\Sigma}_{m}} + \frac{1}{\Omega^{\Sigma''}_{m''} + \varepsilon_{n'} - \varepsilon_n} \right) \hat{c}'^{+}_{a'm'} \hat{b}^{+}_{c'n'} \hat{c}_{am} \hat{b}_{cn} \end{aligned} \tag{D29b}$$

They translate into the formalism of first quantization as

$$V_{12}=\frac{\alpha_s}{R}\mathbf{F}_1\cdot\mathbf{F}_2\sum_{J=0}^{\infty}\mu_{12}(J)K_{12}(J) \tag{D30}$$

The generators in the scalar product $\mathbf{F}_1\cdot\mathbf{F}_2$ are again understood to act in the representations of $SU(3)_{\text{color}}$ that are appropriate to the different particles [see equations (D5) and (D23)].

The operator $K_{12}(J)$ has the matrix elements

$$\langle\Sigma_1'm_1',n_2'|K_{12}(J)|\Sigma_1 m_1,n_2\rangle$$

$$=(-1)^{J+1}[\tfrac{3}{2}(J+2)]^{1/2}\sum_M(-1)^{M+M'}F_{JM}(n_2',n_2)\begin{pmatrix}J' & J & J\\ -M' & -M & M\end{pmatrix} \tag{D31a}$$

$$\langle\Sigma_1'm_1',\bar{n}_2'|K_{12}(J)|\Sigma_1 m_1,\bar{n}_2\rangle$$

$$=-[\tfrac{3}{2}(J+2)]^{1/2}\sum_M(-1)^{M+M'}F_{JM}(-\bar{n}_2,-\bar{n}_2')\begin{pmatrix}J' & J & J\\ -M' & -M & M\end{pmatrix} \tag{D31b}$$

and the operator $\mu_{12}(J)$ is given by

$$\langle\Sigma_1'm_1',n_2'|\mu_{12}(J)|\Sigma_1 m_1,n_2\rangle$$

$$=\frac{(-1)^{J+1}}{[24(J+2)]^{1/2}}\sum_{N\Sigma}\frac{\eta_{\Sigma_1'}\eta_{\Sigma}g^{\Sigma\Sigma}}{R^3\Omega_m^{\Sigma}(\Omega_{m_1}^{\Sigma_1}\Omega_{m_1'}^{\Sigma_1'})^{1/2}}$$

$$\times S^{\Sigma}_{n_2'n_2m}U^{\Sigma_1\Sigma_1'\Sigma}_{m_1m_1'm}(-,+)\left(\frac{1}{\Omega_m^{\Sigma}-\Omega_{m_1}^{\Sigma_1}+\Omega_{m_1'}^{\Sigma_1'}}+\frac{1}{\Omega_m^{\Sigma}-\varepsilon_{n_2}+\varepsilon_{n_2'}}\right) \tag{D32a}$$

$$\langle\Sigma_1'm_1',\bar{n}_2'|\mu_{12}(J)|\Sigma_1 m_1,\bar{n}_2\rangle$$

$$=\frac{(-1)}{[24(J+2)]^{1/2}}\sum_{N\Sigma}\frac{\eta_{\Sigma_1'}\eta_{\Sigma}g^{\Sigma\Sigma}}{R^3\Omega_m^{\Sigma}(\Omega_{m_1}^{\Sigma_1}\Omega_{m_1'}^{\Sigma_1'})^{1/2}}$$

$$\times S^{\Sigma}_{-\bar{n}_2-\bar{n}_2'm}U^{\Sigma_1\Sigma_1'\Sigma}_{m_1m_1'm}(-,+)\left(\frac{1}{\Omega_m^{\Sigma}-\Omega_{m_1}^{\Sigma_1}+\Omega_{m_2'}^{\Sigma_2'}}+\frac{1}{\Omega_m^{\Sigma}-\varepsilon_{\bar{n}_2}+\varepsilon_{\bar{n}_2'}}\right) \tag{D32b}$$

Here, the cases (a) and (b) refer to the quark-gluon and antiquark-gluon systems, respectively.

D3. Systems Consisting of Gluons

There are three different contributions to the gluon-gluon interaction in second-order perturbation theory: the one-gluon exchange, the annihilation into a gluon, and the four-gluon vertex, which are shown in Figures 2k, 2l, and 2m, respectively. The first two of these interactions emerge from the part of the interaction Hamiltonian that is cubic in the gluon field operators. Evaluating the matrix elements $V_{k'k}$ in equation (5.3) with state vectors of the form

$$|\hat{\Phi}_k\rangle = \hat{c}^{\Sigma_1 +}_{a_1 m_1} \hat{c}^{\Sigma_2 +}_{a_2 m_2} |\hat{0}\rangle \tag{D33}$$

and considering terms with one contraction of the gluon fields in the Wick expansion of the time-ordered product, we are still left with the sum of the gluon exchange and the annihilation interaction.

The contribution due to the gluon exchange is easily separated, yielding the two-body operator

$$\begin{aligned}\hat{V} = \frac{g^2}{4\pi R}(-if_{ba'''a'})(-if_{ba''a}) \frac{\eta_{\Sigma'''}\eta_{\Sigma''}\eta_{\tilde{\Sigma}} g^{\tilde{\Sigma}\Sigma}}{8R^4\Omega^{\tilde{\Sigma}}_{\tilde{m}}(\Omega^{\Sigma}_{m}\Omega^{\Sigma'}_{m'}\Omega^{\Sigma''}_{m''}\Omega^{\Sigma'''}_{m'''})^{1/2}}(-1)^{M'''+M''+\tilde{M}} \\ \times \begin{pmatrix} J''' & \tilde{J} & J \\ -M''' & \tilde{M} & M \end{pmatrix} \begin{pmatrix} J'' & \tilde{J} & J \\ -M'' & -\tilde{M} & M \end{pmatrix} \frac{U^{\Sigma'''\tilde{\Sigma}\Sigma'}_{m'''\tilde{m}m'}(-,+)\, U^{\Sigma''\tilde{\Sigma}\Sigma}_{m''\tilde{m}m}(-,+)}{\Omega^{\tilde{\Sigma}}_{\tilde{m}} + \Omega^{\Sigma''}_{m''} - \Omega^{\Sigma}_{m}} \\ \times \hat{c}^{\Sigma'''+}_{a'''m'''}\hat{c}^{\Sigma''+}_{a''m''}\hat{c}^{\Sigma'}_{a'm'}\hat{c}^{\Sigma}_{am}\end{aligned} \tag{D34}$$

For the annihilation, we arrive in a similar fashion at

$$\begin{aligned}\hat{V} = \frac{g^2}{4\pi R}(-if_{ba'''a''})(-if_{ba'a}) \frac{\eta_{\Sigma'''}\eta_{\Sigma''}\eta_{\tilde{\Sigma}} g^{\tilde{\Sigma}\Sigma}}{32R^4\Omega^{\tilde{\Sigma}}_{\tilde{m}}(\Omega^{\Sigma}_{m}\Omega^{\Sigma'}_{m'}\Omega^{\Sigma''}_{m''}\Omega^{\Sigma'''}_{m'''})^{1/2}}(-1)^{M'''+M''+\tilde{M}} \\ \times \begin{pmatrix} J''' & \tilde{J} & J'' \\ -M''' & -\tilde{M} & -M'' \end{pmatrix} \begin{pmatrix} J' & \tilde{J} & J \\ M' & \tilde{M} & M \end{pmatrix} U^{\Sigma'''\tilde{\Sigma}\Sigma''}_{m'''\tilde{m}m''}(-,-)\, U^{\Sigma'\tilde{\Sigma}\Sigma}_{m'\tilde{m}m}(+,+) \\ \times \left(\frac{1}{\Omega^{\tilde{\Sigma}}_{\tilde{m}} + \Omega^{\Sigma'''}_{m'''} + \Omega^{\Sigma'''}_{m''}} + \frac{1}{\Omega^{\tilde{\Sigma}}_{\tilde{m}} - \Omega^{\Sigma'}_{m'} - \Omega^{\Sigma}_{m}}\right) \hat{c}^{\Sigma'''+}_{a'''m'''}\hat{c}^{\Sigma''+}_{a''m''}\hat{c}^{\Sigma'}_{a'm'}\hat{c}^{\Sigma}_{am}\end{aligned} \tag{D35}$$

Here, we have made use of the function $U^{\Sigma_1\Sigma_2\Sigma_3}_{m_1m_2m_3}(\sigma_1, \sigma_2)$ defined in equations (D27) and (D28). In first quantization, the corresponding two-body operator V_{12} is given by

$$V_{12} = \frac{\alpha_s}{R}\mathbf{F}_1 \cdot \mathbf{F}_2 \sum_{J=0}^{\infty} \mu_{12}(J) K_{12}(J) \tag{D36}$$

This form applies to both the gluon exchange and the annihilation interaction. Of course, $\mathbf{F}_i (i = 1, 2)$ denotes the vector of the eight generators of $SU(3)_{\text{color}}$ in the adjoint representation, which are connected to the structure constants by equation (D23). The angular momentum structure of V_{12} is contained in $K_{12}(J)$. For the gluon exchange, $K_{12}(J)$ is given by

$$\begin{aligned}&\langle \Sigma_1' m_1', \Sigma_2' m_2' | K_{12}(J) | \Sigma_1 m_1, \Sigma_2 m_2 \rangle \\ &= 2(2J+1) \sum_{M} (-1)^{M_1'+M_2'+M} \\ &\times \begin{pmatrix} J_1' & J & J_1 \\ -M_1' & M & M_1 \end{pmatrix} \begin{pmatrix} J_2' & J & J_2 \\ -M_2' & -M & M_2 \end{pmatrix}\end{aligned} \tag{D37}$$

while for the annihilation we have

$$\langle \Sigma_1' m_1', \Sigma_2' m_2' | K_{12}(J) | \Sigma_1 m_1, \Sigma_2 m_2 \rangle$$
$$= 2(2J+1) \sum_M (-1)^{M_1'+M_2'+M}$$
$$\times \begin{pmatrix} J_1' & J & J_2' \\ -M_1' & -M & -M_2' \end{pmatrix} \begin{pmatrix} J_1 & J & J_2 \\ M_1 & M & M_2 \end{pmatrix} \tag{D38}$$

Here, the ket $|\Sigma_1 m_1, \Sigma_2 m_2\rangle$ denotes the direct product state of two gluons with polarizations Σ_1 and Σ_2 and quantum numbers m_1 and m_2, respectively. The matrix elements of the operators $\mu_{12}(J)$ are

$$\langle \Sigma_1' m_1', \Sigma_2' m_2' | \mu_{12}(J) | \Sigma_1 m_1, \Sigma_2 m_2 \rangle$$
$$= \frac{1}{16(2J+1)} \frac{\eta_{\Sigma_1'} \eta_{\Sigma_2'}}{R^4 (\Omega^{\Sigma_1}_{m_1} \Omega^{\Sigma_2}_{m_2} \Omega^{\Sigma_1'}_{m_1'} \Omega^{\Sigma_2'}_{m_2'})^{1/2}}$$
$$\times \sum_{N\Sigma} \frac{\eta_\Sigma g^{\Sigma\Sigma}}{\Omega^\Sigma_m} U^{\Sigma_1'\Sigma\Sigma_1}_{m_1' m m_1}(-,+) U^{\Sigma_2'\Sigma\Sigma_2}_{m_2' m m_2}(-,+)$$
$$\times \left(\frac{1}{\Omega^\Sigma_m + \Omega^{\Sigma_1'}_{m_1'} - \Omega^{\Sigma_1}_{m_1}} + \frac{1}{\Omega^\Sigma_m + \Omega^{\Sigma_2'}_{m_2'} - \Omega^{\Sigma_2}_{m_2}} \right) \tag{D39}$$

for the gluon exchange and

$$\langle \Sigma_1' m_1', \Sigma_2' m_2' | \mu_{12}(J) | \Sigma_1 m_1, \Sigma_2 m_2 \rangle$$
$$= \frac{1}{16(2J+1)} \frac{1}{R^4 (\Omega^{\Sigma_1}_{m_1} \Omega^{\Sigma_2}_{m_2} \Omega^{\Sigma_1'}_{m_1'} \Omega^{\Sigma_2'}_{m_2'})^{1/2}}$$
$$\times \sum_{N\Sigma} \frac{\eta_\Sigma g^{\Sigma\Sigma}}{\Omega^\Sigma_m} U^{\Sigma_1'\Sigma\Sigma_2'}_{m_1' m m_2'}(-,-) U^{\Sigma_1\Sigma\Sigma_2}_{m_1 m m_2}(+,+)$$
$$\times \left(\frac{1}{\Omega^\Sigma_m + \Omega^{\Sigma_1'}_{m_1'} + \Omega^{\Sigma_2'}_{m_2'}} + \frac{1}{\Omega^\Sigma_m - \Omega^{\Sigma_1}_{m_1} - \Omega^{\Sigma_2}_{m_2}} \right) \tag{D40}$$

for the annihilation.

We now turn to the gluon–gluon interaction, which is described by the elementary four-gluon vertex, Figure 2m. The source of this process is the part of the interaction Hamiltonian that is proportional to g^2. Therefore, the contribution in second-order perturbation theory to the matrix element $V_{k'k}$ is found in the first term of equation (5.3). Using state vectors of the form (D33), we arrive at the two-body operator

$$\hat{V}=\frac{1}{8}g^2\frac{1}{(\Omega^{\Sigma}_{m}\Omega^{\Sigma'}_{m'}\Omega^{\Sigma''}_{m''}\Omega^{\Sigma'''}_{m'''})^{1/2}}(-if_{ba'''a'})(-if_{ba''a})\eta_{\Sigma'''}\eta_{\Sigma''}(-1)^{M'''+M''}$$
$$\times(2F^{\Sigma'''\Sigma\Sigma''\Sigma'}_{\overset{*}{m}'''m\overset{*}{m}''m'}-F^{\Sigma'''\Sigma'\Sigma''\Sigma}_{\overset{*}{m}'''m'\overset{*}{m}''m}-F^{\Sigma'''\Sigma''\Sigma'\Sigma}_{\overset{*}{m}'''\overset{*}{m}''m'm})\hat{c}^{\Sigma'''+}_{a'''m'''}\hat{c}^{\Sigma''+}_{a''m''}\hat{c}^{\Sigma'}_{a'm'}\hat{c}^{\Sigma}_{am} \quad \text{(D41)}$$

The integral $F^{\Sigma\Sigma'\Sigma''\Sigma'''}_{mm'm''m'''}$ is given in Appendix C, equation (C26).

In order to obtain a factorization of the angular momentum contribution in $\hat{V}$, we decompose $F^{\Sigma\Sigma'\Sigma''\Sigma'''}_{mm'm''m'''}$ as follows:

$$F^{\Sigma\Sigma'\Sigma''\Sigma'''}_{mm'm''m'''}$$
$$=\frac{1}{R^3}\sum_{lk}\frac{(2l+1)(-1)^k}{4\pi}\begin{pmatrix}J & l & J'\\ M & k & M'\end{pmatrix}\begin{pmatrix}J'' & l & J'''\\ M'' & -k & M'''\end{pmatrix}T^{\Sigma\Sigma'\Sigma''\Sigma'''}_{mm'm''m'''}(l) \quad \text{(D42)}$$

The form of $T^{\Sigma\Sigma'\Sigma''\Sigma'''}_{mm'm''m'''}(l)$ is easily determined from equations (C30) and (C31). In first quantization, the corresponding two-body operator V_{12} is given by

$$V_{12}=\frac{\alpha_s}{R}\mathbf{F}_1\cdot\mathbf{F}_2\sum_{l=0}^{\infty}[\mu^A_{12}(l)K^A_{12}(l)+\mu^B_{12}(l)K^B_{12}(l)+\mu^C_{12}(l)K^C_{12}(l)] \quad \text{(D43)}$$

The $\mathbf{F}$ is again the vector of the $SU(3)_{\text{color}}$ generators in the eight-dimensional adjoint representation. The matrix elements of the $K_{12}(l)$ operators are

$$\langle\Sigma'_1m'_1,\Sigma'_2m'_2|K^A_{12}(l)|\Sigma_1m_1,\Sigma_2m_2\rangle$$
$$=2(2l+1)\sum_M(-1)^{M'_1+M'_2+M}$$
$$\times\begin{pmatrix}J'_1 & l & J_1\\ -M'_1 & M & M_1\end{pmatrix}\begin{pmatrix}J'_2 & l & J_2\\ -M'_2 & -M & M_2\end{pmatrix} \quad \text{(D44a)}$$

$$\langle\Sigma'_1m'_1,\Sigma'_2m'_2|K^B_{12}(l)|\Sigma_1m_1,\Sigma_2m_2\rangle$$
$$=2(2l+1)\sum_M(-1)^{M'_1+M'_2+M}$$
$$\times\begin{pmatrix}J'_1 & l & J'_2\\ -M'_1 & -M & -M'_2\end{pmatrix}\begin{pmatrix}J_1 & l & J_2\\ M_1 & M & M_2\end{pmatrix} \quad \text{(D44b)}$$

$$\langle\Sigma'_1m'_1,\Sigma'_2m'_2|K^C_{12}(l)|\Sigma_1m_1,\Sigma_2m_2\rangle$$
$$=2(2l+1)\sum_M(-1)^{M'_1+M'_2+M}$$
$$\times\begin{pmatrix}J'_1 & l & J_2\\ -M'_1 & M & M_2\end{pmatrix}\begin{pmatrix}J'_2 & l & J_1\\ -M'_2 & -M & M_1\end{pmatrix} \quad \text{(D44c)}$$

The $\mu_{12}(l)$ operators are given in terms of the T-function of equation (D42) as

$$\langle \Sigma_1' m_1', \Sigma_2' m_2' | \mu_{12}^A(l) | \Sigma_1 m_1, \Sigma_2 m_2 \rangle$$

$$= -\frac{\eta_{\Sigma_1'}\eta_{\Sigma_2'}}{8R^2(\Omega_{m_1}^{\Sigma_1}\Omega_{m_2}^{\Sigma_2}\Omega_{m_1'}^{\Sigma_1'}\Omega_{m_2'}^{\Sigma_2'})^{1/2}} T_{m_1' m_1 m_2' m_2}^{\Sigma_1'\Sigma_1\Sigma_2'\Sigma_2}(l) \tag{D45a}$$

$$\langle \Sigma_1' m_1', \Sigma_2' m_2' | \mu_{12}^B(l) | \Sigma_1 m_1, \Sigma_2 m_2 \rangle$$

$$= -\frac{\eta_{\Sigma_1'}\eta_{\Sigma_2'}}{8R^2(\Omega_{m_1}^{\Sigma_1}\Omega_{m_2}^{\Sigma_2}\Omega_{m_1'}^{\Sigma_1'}\Omega_{m_2'}^{\Sigma_2'})^{1/2}} T_{m_1' m_2' m_1 m_2}^{\Sigma_1'\Sigma_2'\Sigma_1\Sigma_2}(l) \tag{D45b}$$

$$\langle \Sigma_1' m_1', \Sigma_2' m_2' | \mu_{12}^C(l) | \Sigma_1 m_1, \Sigma_2 m_2 \rangle$$

$$= -\frac{\eta_{\Sigma_1'}\eta_{\Sigma_2'}}{8R^2(\Omega_{m_1}^{\Sigma_1}\Omega_{m_2}^{\Sigma_2}\Omega_{m_1'}^{\Sigma_1'}\Omega_{m_2'}^{\Sigma_2'})^{1/2}} T_{m_1' m_2 m_2' m_1}^{\Sigma_1'\Sigma_2\Sigma_2'\Sigma_1}(l) \tag{D45c}$$

D4. Special Cases

Let us now consider the interactions between particles with the lowest possible angular momentum, i.e., $\kappa = \pm 1 (j = \frac{1}{2})$ for quarks and $J = 1$ for the (transverse) gluons. Here, the two-body operators K_{12} given in Sections D1–D3 can easily be expressed in terms of products of one-body spin operators $\mathbf{S}$. Using the Wigner–Eckart theorem, we obtain the matrix elements of these one-body operators in spherical (instead of Cartesian) coordinates as

$$(S^k)_{m'm} = \begin{cases} (-1)^{m'-1/2}\sqrt{\frac{3}{2}}\begin{pmatrix} \frac{1}{2} & 1 & \frac{1}{2} \\ -m' & k & m \end{pmatrix} & \text{for quarks} \quad \text{(D46a)} \\ -(-1)^{m+1/2}\sqrt{\frac{3}{2}}\begin{pmatrix} \frac{1}{2} & 1 & \frac{1}{2} \\ m & k & -m' \end{pmatrix} & \text{for antiquarks} \quad \text{(D46b)} \end{cases}$$

$$(S^k)_{M'M} = (-1)^{M'-1}\sqrt{6}\begin{pmatrix} 1 & 1 & 1 \\ -M' & k & M \end{pmatrix} \qquad \text{for gluons} \tag{D46c}$$

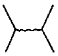
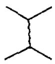
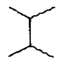
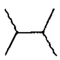
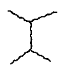
The scalar product in spherical coordinates is given by

$$\mathbf{S}_1 \cdot \mathbf{S}_2 = \sum_{k=-1}^{1} (-1)^k S_1^k S_2^{-k} \tag{D47}$$

With the help of equations (D46) and (D47), the two-body operators K_{12} may be rewritten as shown in Table IX, where we use the abbreviation

$$S_{12} = \mathbf{S}_1 \cdot \mathbf{S}_2 \tag{D48}$$

Table IX. The Two-Body Operators $K_{12}(J)$ that Contain the Spin Structure of the Various Diagrams[a]

Diagram	$K_{12}(0)$	$K_{12}(1)$	$K_{12}(2)$
	1	$4S_{12}$	0
	$\frac{1}{4}-S_{12}$	$\frac{3}{4}+S_{12}$	0
	—	$\frac{1}{2}-S_{12}$	$1+S_{12}$
	—	$\frac{1}{2}+S_{12}$	$1-S_{12}$
	1	S_{12}	0
	$\frac{2}{3}$	S_{12}	$-\frac{8}{3}+S_{12}+2S_{12}^2$
	$-\frac{2}{3}+\frac{2}{3}S_{12}^2$	$-2+S_{12}+S_{12}^2$	$\frac{2}{3}+S_{12}+\frac{1}{3}S_{12}^2$
 A	$\frac{2}{3}$	S_{12}	$-\frac{8}{3}+S_{12}+2S_{12}^2$
B	$-\frac{2}{3}+\frac{2}{3}S_{12}^2$	$-2+S_{12}+S_{12}^2$	$\frac{2}{3}+S_{12}+\frac{1}{3}S_{12}^2$
C	$-\frac{2}{3}+\frac{2}{3}S_{12}+\frac{2}{3}S_{12}^2$	$2-S_{12}^2$	$\frac{2}{3}-\frac{2}{3}S_{12}+\frac{1}{3}S_{12}^2$

[a]The quarks and gluons in the initial and final states occupy the lowest angular momentum states, $j=\frac{1}{2}$ for quarks and $J=1$ for gluons. Note that $S_{12}=\mathbf{S}_1\cdot\mathbf{S}_2$.

Due to the conservation of angular momentum at the vertices, only the first two or three terms in the sums that occur in the expressions for the two-body operators V_{12} are nonzero. Using the results of Table IX, we arrive in a straightforward way at the dimensionless interaction operators, which are connected to the V_{12} by

$$V_{12}=(g^2/4\pi R)\Delta_{12} \tag{D49}$$

for the various Feynman diagrams as shown in Table I. Note that we have rearranged the terms in the gluon–gluon interaction in order to obtain the same form of Δ_{12} for the three different diagrams. Accordingly, we use here the operators ρ_{12}, which are certain linear combinations of the μ_{12}. In order not to overburden the formulas, we use also the shorthand

$$F_{12}=\mathbf{F}_1\cdot\mathbf{F}_2, \qquad T_{12}=\mathbf{T}_1\cdot\mathbf{T}_2 \tag{D50}$$

for the scalar product of the $SU(3)_{\text{color}}$ and $SU(2)_{\text{isospin}}$ generators, respectively.

The matrix elements of the two-body operators μ_{12} and ρ_{12} are given in Tables II–IV. They correspond to the lowest energy cavity states, i.e.,

$$R\varepsilon_n = \begin{cases} 2.042787 & \kappa = -1 \\ 3.811539 & \kappa = +1 \end{cases} \qquad \text{massless quarks}$$

$$R\Omega_m^{\Sigma} = \begin{cases} 2.743707 & \Sigma = \mathcal{M} \\ 4.493409 & \Sigma = \mathcal{E} \end{cases} \qquad \text{gluons} \tag{D51}$$

In the quark–quark and gluon–gluon cases, where two different direct product states can have the same (asymptotic) energy, we use the (anti) symmetrized states

$$(-, +)_{\pm} = \frac{1}{\sqrt{2}}[(-, +) \pm (+, -)] \tag{D52}$$

for the quarks and

$$(\mathcal{M}, \mathcal{E})_{\pm} = \frac{1}{\sqrt{2}}[\mathcal{M}, \mathcal{E}) \pm (\mathcal{E}, \mathcal{M})] \tag{D53}$$

for the gluons in an obvious notation.

APPENDIX E. CONVENTIONS AND UNITS

We use the flat Minkowski space metric $g^{\mu\nu}$ with the signature

$$g^{\mu\nu} = g_{\mu\nu} = \text{diag}(1, -1, -1, -1), \qquad g^{\mu}_{\nu} = \delta^{\mu}_{\nu} \tag{E1}$$

Greek indices ($\mu, \nu, \ldots$) can take the values 0, 1, 2, and 3, Latin indices ($k, l, \ldots$) the values 1, 2, and 3 when they refer to space-time. They are raised or lowered with $g^{\mu\nu}$,

$$\begin{aligned} x^{\mu} &= g^{\mu\nu}x_{\nu}, \qquad & x_{\mu} &= g_{\mu\nu}x^{\nu} \\ x_0 &= x^0, \qquad & x_k &= -x^k = -[\mathbf{x}]^k \end{aligned} \tag{E2}$$

The 4×4 Dirac γ matrices that satisfy the Clifford algebra

$$\{\gamma^{\mu}, \gamma^{\nu}\} = 2g^{\mu\nu} \tag{E3}$$

may be represented as follows:

$$\gamma^0 = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix}, \qquad \gamma^k = \begin{pmatrix} 0 & \sigma^k \\ -\sigma^k & 0 \end{pmatrix} \tag{E4}$$

where the σ^k are the 2×2 Pauli matrices

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{E5}$$

Under Hermitian conjugation, we have

$$(\sigma^k)^+ = \sigma^k, \qquad (\gamma^\mu)^+ = \gamma^0\gamma^\mu\gamma^0 = \gamma_\mu \tag{E6}$$

We employ the usual $3j$-, $6j$-, $9j$-symbols, vector spherical harmonics, and spherical spinors as defined by Edmonds (1957).

Throughout the text, we use "natural" units with

$$\hbar = c = 1 \tag{E7}$$

The correct factors of $\hbar$ and c, which have to be supplied in the various formulas, are easily found by considering the dimensions. As an example, equations (A12) and (A14) should be replaced by

$$\omega_n = \varepsilon_n R/\hbar c \qquad \text{and} \qquad \zeta_f = m_f Rc/\hbar \tag{E8}$$

Some useful numerical relations are

$$\begin{aligned} \hbar c &= 197.3285851 \text{ MeV fm} \\ 1 \text{ GeV} &= 5.06768963 \hbar c \text{ fm}^{-1} \end{aligned} \tag{E9}$$

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